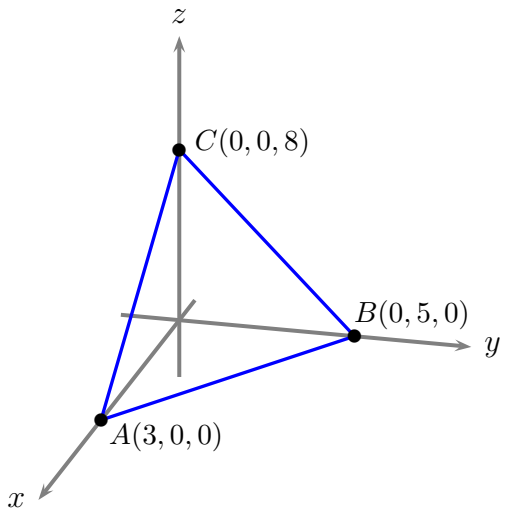
Distance from a Point to a Plane



Example 1. Let T be the plane passing through the points A, B, and C as shown in the figure above and answer the questions below.

a. Find the equation of T.

Let $\mathbf{u} = \overrightarrow{AB} = \langle -3, 5, 0\rangle$ and $\mathbf{v} = \overrightarrow{AC} = \langle -3, 0, 8\rangle$. In class we showed that $\mathbf{v} \times \mathbf{u} = \langle 40, 24, 15\rangle$. It follows that the equation for T is given by

$$40(x-3) + 24y + 15z = 0$$

or, using the **equivalent** intercept form

$$\frac{x}{3} + \frac{y}{5} + \frac{z}{8} = 1$$

b. Find the area of the triangle ΔABC.

This one is easy.

$$\text{Area} = \frac{|\mathbf{v} \times \mathbf{u}|}{2} = \frac{\sqrt{40^2 + 24^2 + 15^2}}{2} = \frac{49}{2}$$

c. Find the distance from T to the origin $O(0, 0, 0)$.

Let d be the distance from O to T, $\mathbf{n} = \mathbf{v} \times \mathbf{u}$, and $\mathbf{w} = \overrightarrow{OA}$. Then $\mathbf{w} = 3\,\mathbf{i}$ and

$$d = |\operatorname{proj}_{\mathbf{n}} \mathbf{w}| = \frac{3\,\mathbf{i} \cdot \mathbf{n}}{\mathbf{n} \cdot \mathbf{n}} \sqrt{\mathbf{n} \cdot \mathbf{n}} = \frac{120}{49}$$

Note: Other choices for $\mathbf{w}$ work as well. For example, we could have chosen $\mathbf{w} = \overrightarrow{OB} = 5\,\mathbf{j}$.