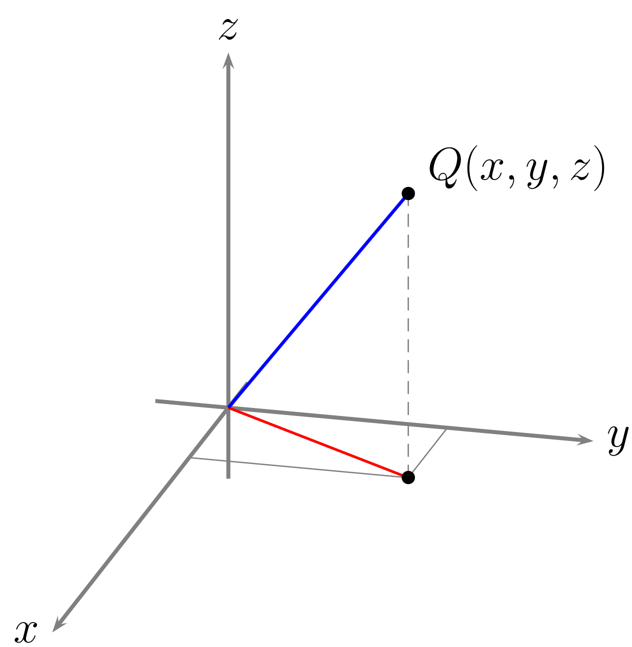
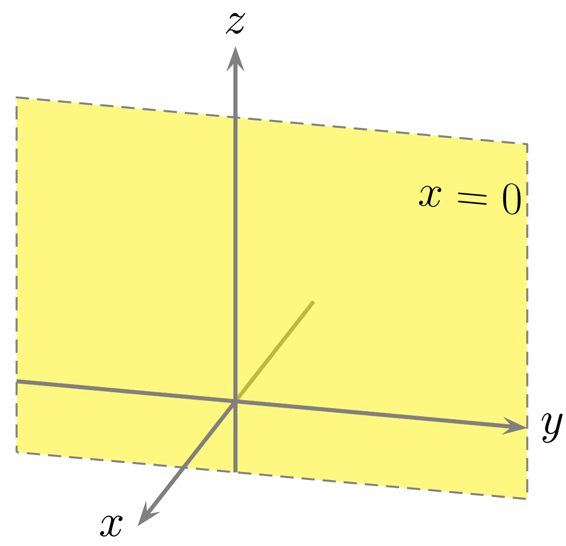
12.1 Three Dimensional Coordinate Systems

We often refer to the xy-coordinate system as the Euclidian plane or simply, the plane. In symbols the plane is denoted $\mathbb{R}^2$. To identify a point in the plane, we use its Cartesian (or rectangular) coordinates. That is, we identify each point in $P \in \mathbb{R}^2$ using the notation $P(x, y)$ or simply (x, y). In this course we will also be working with the three-dimensional analogue of the plane, $\mathbb{R}^3$ or **three-space**.

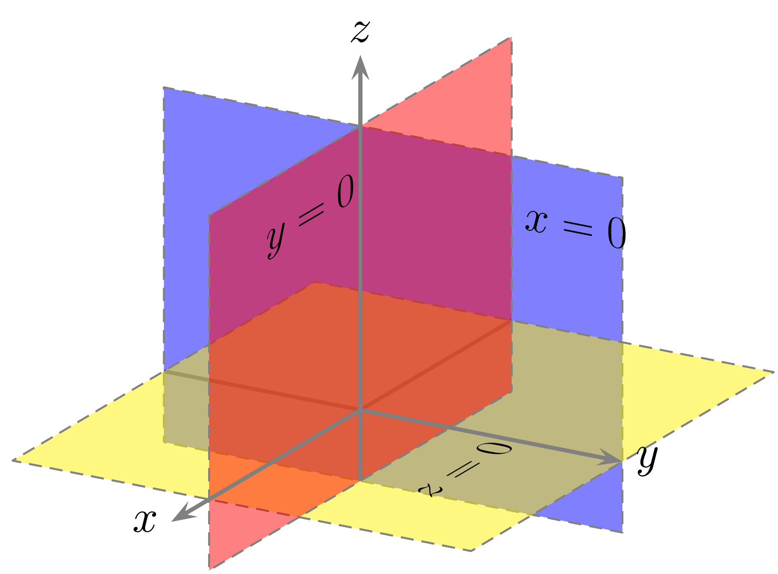


The above sketch shows a (right-handed) three-dimensional coordinate system. Here the ordered triple (x, y, z) gives the coordinates of a point Q as indicated in the sketch (the red and blue lines are artificial and have been included to give the reader a sense of perspective).

The sketch below shows the infinite plane $x = 0$.



In a similar manner one can sketch the graphs of the equations $y = 0$ and $z = 0$.



Notice that these three planes break up three-space into eight *octants*. The **first octant** coincides with positive x, y and z-coordinates and is three-dimensional analogue of quadrant I in the plane.

Distance and Spheres in Space

Proposition 1. The Distance Formula

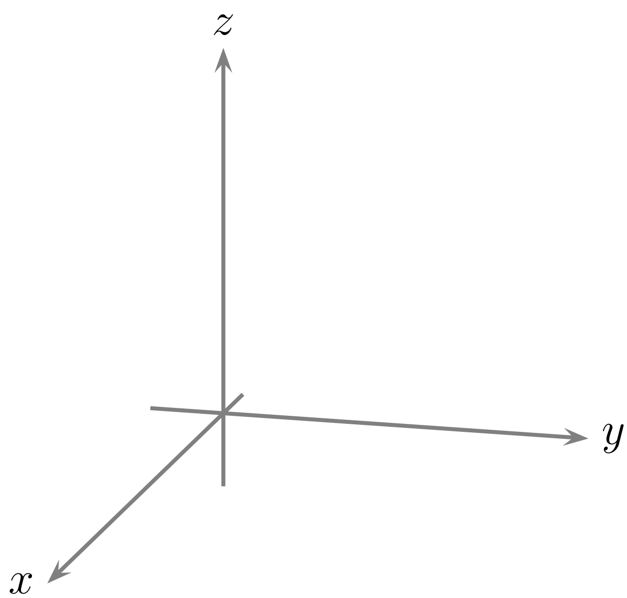
Let $P_1\,(x_1, y_1, z_1)$ and $P_2\,(x_2, y_2, z_2)$ be points in space (i.e., $P_1, P_2 \in \mathbb{R}^3$). The distance between P_1 and P_2 is

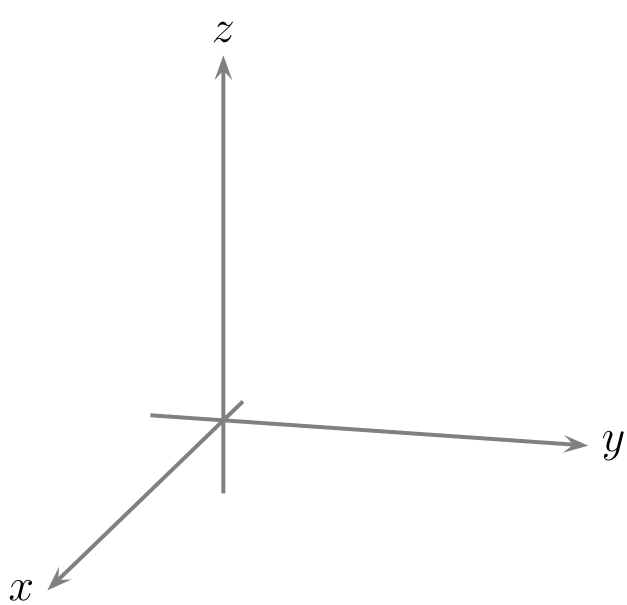
$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{1}$$

Proof. Apply the Pythagorean Theorem twice.

It follows from the distance formula given above that the equation of a sphere of radius a centered at (x_0, y_0, z_0) is given by

(2) $$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$


z
y
x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