Name: ____________________ PID: ______________ Section: ______

Instructions. Grading is based on method. SHOW ALL WORK.

1. Suppose that $b_n \geq 0$ for each $n \in \mathbb{N}$ and that $\lim_{n\to\infty} nb_n = 0$.

   (a) (5 points) Show that

   $$\lim_{n\to\infty}(1+b_n)^n = 1 \tag{1}$$

   Note: l'Hôpital's rule is of no help here.

   (b) (5 points) Show that

   $$\lim_{n\to\infty}(1-b_n)^n = 1$$

   Hint: Let $a_n = b_n(1-b_n)^{-1}$. First observe that $\lim_{n\to\infty} na_n = 0$. Since $a_n \geq 0$ for n sufficiently large, we can use part (1a) on the sequence $(1+a_n)^n$.

2. For each $k, n \in \mathbb{N}$, let $f_n(x,k) = \dfrac{\sqrt{n}x^k}{1+nx^2}, \quad x \in I = [0,1]$.

   (a) (5 points) Show that $f_n(\cdot, 1) \to 0$ uniformly on I. That is, show that $\dfrac{\sqrt{n}x^1}{1+nx^2} \to 0$ uniformly on $[0,1]$.

   (b) For each $k \in \mathbb{N}$, show that $f_n(\cdot, k) \to 0$ uniformly on I. (*Hint:* Part (a) may help.)