Show that if $\{a_n\}$ is a decreasing sequence of real numbers and if $\sum a_n$ converges, then $\lim_{n\to\infty} na_n = 0$. (Notice that the two assumptions guarantee that $a_n \searrow 0$ as $n \to \infty$. In particular, $a_n > 0$.) Here's a better hint than the one given in the text.

Hint: Suppose this is false. That is, suppose that $\lim_{n\to\infty} na_n = c > 0$. Now use the Comparison Test to conclude that the series $\sum a_n$ diverges.

Below I put together a proof based on the textbook's hint.

Proof. Let $\varepsilon > 0$. Then following the textbook's hint (the Cauchy Criterion), there exists an $M \in \mathbb{N}$ such that for all $n > M$ we have

$$a_{M+1} + a_{M+2} + \cdots + a_n < \frac{\varepsilon}{2} \tag{1}$$

Note: Absolute value signs are not necessary since the terms are positive. Now we fix M for the remainder of this argument. By Corollary 14.5 from the text, there exists a $P \in \mathbb{N}$ such that $n \geq P$ implies $a_n < \varepsilon/(2M)$.

Now let $N = \max\{M, P\}$ and let $n > N$. Then

$$na_n = \underbrace{a_n + a_n + \cdots + a_n}_{n \text{ terms}} \tag{2}$$

$$\leq \underbrace{a_P + a_P + \cdots + a_P}_{M \text{ terms}} + \underbrace{a_{M+1} + a_{M+2} + \cdots + a_n}_{n-M \text{ terms}} \tag{3}$$

$$< M\, a_P + \frac{\varepsilon}{2} \tag{4}$$

$$< M\, \frac{\varepsilon}{2M} + \frac{\varepsilon}{2} = \varepsilon \tag{5}$$

(3) - Since the sequence is decreasing. That is, since $a_n \leq a_{n-1} \leq \cdots \leq a_1$.
(4) - From (1).
(5) - Since $a_n \to 0$ as $n \to \infty$. □

Remark. Notice how monotonicity is used in a big way!