1. (5 points) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of positive terms. The review notes describe 6 distinct convergence tests for the infinite series $\sum_{n=1}^{\infty} a_n$. Name 5 of them.

   i. Divergence Test
   ii. Ratio Test
   iii. Root Test
   iv. Direct Comparison Test
   v. Limit Comparison Test
   vi. Cauchy Condensation Test (not the Cauchy Comparison Test)

2. (10 points) *Carefully* choose one of the convergence tests from the list above and describe it completely. *Don't forget to include additional hypotheses if necessary.*

   Solution:

   See `www.math.msu.edu/~hensh/courses/320/summer15/handouts/ConvTests-2up.pdf`.

3. (10 points) Decide whether each of the following series converge or diverge. *Justify your claim.*

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(1/n)}{n^2}$

   Solution:

   We claim the series converges. To see this, let $b_n = 1/n^2$ and note that $\sum b_n < \infty$ since this is a p-series with $p = 2 > 1$. Now $0 < \cos x \leq 1$ for all $x \in [0, 1]$, so that

   $$a_n = \frac{\cos(1/n)}{n^2} \leq \frac{1}{n^2} = b_n \tag{1}$$

   And the result now follows by the Direct Comparison Test. Note that we could also use the Limit Comparison Test. A few comments are in order.

   i. It is useless to conclude that $\frac{\cos(1/n)}{n^2}$ converges. In fact, it converges to zero, but that tells us nothing. We *are* interested in the convergence (or divergence) of $\sum_{n=1}^{\infty} \frac{\cos(1/n)}{n^2}$.

   ii. To be precise in (1), we appeal to the order properties of $\mathbb{R}$. Specifically, since $1/n^2 > 0$ property **O5** yields

   $$0 < \cos(1/n) \leq 1 \Longrightarrow 0 < \frac{1}{n^2} \cdot \cos(1/n) \leq \frac{1}{n^2} \cdot 1$$

   (b) $\displaystyle\sum_{n=1}^{\infty} a_n$, where $a_n > 0$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} n a_n = 1$.

   Solution:

   We claim that the series diverges. To see this let $b_n = 1/n$, then

   $$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{a_n}{1/n} = \lim_{n\to\infty} n a_n = 1 < \infty$$

   So by the Limit Comparison Test, the series $\sum a_n$ and $\sum b_n$ both converge or they both diverge. But the latter is the well-known harmonic series, which is known to diverge. It is worth noting that the given limit condition guarantees that $a_n \to 0$ as $n \to \infty$. *Can you prove this?*