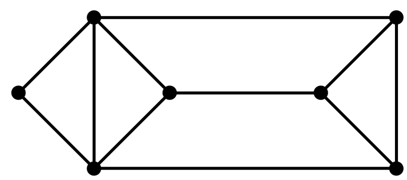
Name: ____________________ PID: ______________ Section: ______

Instructions. Grading is based on method. SHOW ALL WORK.

Please submit your solutions at the beginning of the Final Exam on Wednesday (2023-12-13). I will not accept an electronic submission.

1. (10 points) Find the chromatic polynomial for the graph G shown below.



2. Let $M([n], k)$ denote the set of all partitions of $[n]$ into k linearly ordered (nonempty) subsets, called blocks. Now let $M(0, 0) = 1$ and as usual, let $M(n, k) = |M([n], k)|$. For example,

$$M([3], 2) = \{\{1/23\}, \{1/32\}, \{2/13\}, \{2/31\}, \{3/12\}, \{3/21\}\}$$

and

$$M([4], 3) = \{\{1/2/34\}, \{1/2/43\}, \{1/3/24\}, \{1/3/42\}, \{1/4/23\}, \{1/4/32\},$$
$$\{2/3/14\}, \{2/3/41\}, \{2/4/13\}, \{2/4/31\}, \{3/4/12\}, \{3/4/21\}\}$$

Notice that we borrow from set partition notation. It follows that $M(3, 2) = 6$ and $M(4, 3) = 12$. Note: The ordering of the blocks is irrelevant.

(a) (5 points) Show that $M(n, n) = 1$ for $n > 0$.

(b) (5 points) Show that $M(n, 1) = n!$ for $n > 0$.

(c) (10 points) Prove the following identity.

$$M(n,k) = M(n-1,k-1) + (n+k-1)M(n-1,k)$$