Fall 2	2023
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Date	Section	Exercises*
11/20	1.1.2	1, 2, 4, 9, 11, 14, 15
$11/22^{*}$	1.1.3	1, 2
11/27	1.3.1	1, 3
11/27	1.3.2	3, 5, 7, 8
11/29	1.3.3	4, 5
$11/29^{*}$	1.3.4	2, 3, 4, 5
$12/01^{*}$	1.6.1	1ae, 2, 4, 5 (see item 9 on page 13 of $\underline{\text{HHM}}$ text)
$12/04^{*}$	1.6.2	5
$12/06^{*}$	1.6.4	1, 2

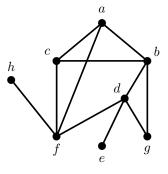


Figure 1: Graph G

11/22 Use the graph G above to answer the questions that follow.

- (a) Sketch the graph $G \setminus d$. Is d a cut vertex? List all of the cut vertices of G.
- (b) Sketch the graph $G \setminus ab$. Is ab a bridge? List all the bridges for G.
- (c) Find the connectivity $\kappa(G)$. Now add eh to E(G) and call the new graph H. What is the connectivity of H?
- (d) Is G complete? Is G regular? Sketch the *complement* of G.

11/29 Prove that if T is a tree, then the average degree of a vertex is strictly less than 2.

12/01 Let T be a tree with max degree $\Delta = \Delta(T)$. Prove that T has at least Δ leaves.

12/01 Use the method outlined in Brooks's theorem to color the graph below. Be sure to specify the sets S_0, S_1, \ldots, S_t and label the corresponding vertices. For colors, use 1, 2, 3, ...

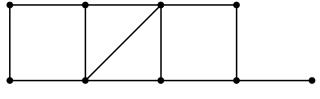
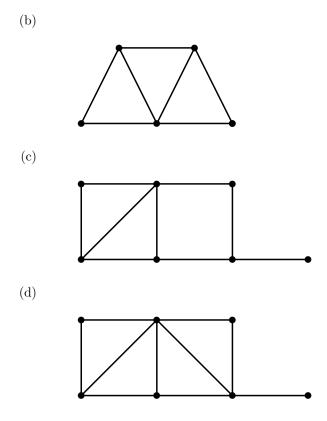


Figure 2: G

- 12/06 Let G be a graph with chromatic polynomial $C_G(x) = x(x-1)^{n-1}$. What kind of graph is G? Prove your claim.
- 12/06 For $n \ge 3$, prove that the chromatic polynomial of the cycle graph C_n is $(x-1)^n + (-1)^n (x-1)$. Hint: Try induction on n.
- 12/06 For $n \ge 4$, prove that the chromatic polynomial of the wheel graph W_n is $x((x-2)^{n-1} (-1)^n(x-2))$. Note: For the base case, $W_4 = K_4$. Also, check out the collection of <u>chromatic polynomials</u> at Wolfram's MathWorld.

- $12/06\,$ Find the chromatic polynomials of the graphs below.
 - (a) A star graph with n vertices.



(e) Notice that G is 3-regular with a cut vertex.

