1. (10 points) A standard deck of 52 playing cards contains 4 suits $\{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$, with 13 cards in each suit. How many cards must be drawn (at random) to guarantee 3 cards from the same suit. *Justify your claim.*

   Solution:

   We must draw 9 cards. Let $r = 2$ and notice that we have $k = 4$ suits. If we draw only 8 cards, we could end up with two $\clubsuit$'s, two $\diamondsuit$'s, two $\heartsuit$'s, and two $\spadesuit$'s. On the other hand, since $9 > 2 \cdot 4$, we must have chosen at least 3 cards in a one suit, by the Pigeonhole Principle.

2. (10 points) Eleven distinct positive integers less than 28 are randomly selected. Prove that at least two of these must always have a common divisor greater than 1.

   Solution:

   Notice that the we can certainly choose the following 6 integers since they are pairwise coprime: 1,11,13,17,19,23.

   Now distribute the remaining integers into four boxes as follows (other choices will also work). The even integers go into box b_1; $3, 9, 27 \in b_2$; $5, 15, 25 \in b_3$; and $7, 21 \in b_4$. Notice that if $m, n \in b_j,\ j \in [4]$, then $\gcd(m, n) > 1$.

   Since we must choose the remaining 5 integers from these four boxes, at least two of the integers must come from the same box by the Pigeonhole Principle. The result follows.