1. (8 points) Let a, b, c be distinct prime numbers and let $n = ab^2c^3$. Excluding $1 = a^0b^0c^0$, how many positive integer divisors does n have? For example, ab and b^2 are two such divisors.

   Solution:

   It turns out that there are 23 divisors.

   To see this note that any divisor of n must be of the form $a^{k_a}b^{k_b}c^{k_c}$, where $0 \leq k_a \leq 1$, $0 \leq k_b \leq 2$, and $0 \leq k_c \leq 3$ are integers. So there are 2 possible exponents for a, 3 possible exponents for b, and 4 possible exponents for c. So by the product rule, the number of possible divisors is $2 \times 3 \times 4$ and since $a^0b^0c^0$ must be excluded, the result follows.

2. (12 points) How many 5 or 6-letter words can be created from the multiset $M = \{\{a, b^2, c^3\}\}$? For example, it is impossible to create a 5-letter word from M using only one c.

   Solution:

   There are $\dfrac{6!}{1!2!3!} = 60$ distinguishable permutations using all six letters.

   We have to work a little harder for the 5-letter words. There are $\frac{5!}{1!2!2!} = 30$ distinguishable permutations using the letters $\{\{a, b^2, c^2\}\}$. There are $\frac{5!}{2!3!} = 10$ distinguishable permutations using the letters $\{\{b^2, c^3\}\}$. Finally, there are $\frac{5!}{1!2!2!} = 20$ distinguishable permutations using the letters $\{\{a, b, c^3\}\}$.

   Since each of these four sets of words are pairwise disjoint, there are $60 + 30 + 10 + 20 = 120$ five or 6-letter words that can be created from M.