# EDITOR'S ENDNOTES

James Fey (University of Maryland), Eric Hart (Maharishi University of Management), Christian Hirsch (Western Michigan University), Harold Schoen (University of Iowa), and Ann Watkins (California State University, Northridge), who are senior curriculum developers for the Core-Plus Mathematics Project, wrote in response to an article about that project:

This letter is a brief response to "A Study of Core-Plus Students Attending Michigan State University" by Hill and Parker (this MONTHLY, December 2006) which claims to provide "compelling" evidence that students using one of the reform high school mathematics curricula are poorly prepared for collegiate mathematics. Although we commend the authors' expressed desire for studies that investigate how students from a reform program in high school do in college mathematics courses, the substantial flaws in the present study render it of little use in exploring this important question. Versions of the Hill and Parker paper, all with the same data from 1996–1999, have been posted on the internet for more than three years, as has a longer response from us that can be found at http://www.wmich.edu/cpmp/cpmpfaq.html#hillparker. We think the conclusions in this paper are neither timely nor valid.

The first problematic aspect of the paper is the fact that nearly all of the students in the Hill and Parker sample used a draft field-test version of a curriculum that no longer exists. Based on our own evaluation, the draft version was revised prior to publication in ways intended to strengthen preparation for college mathematics. The changes made and some of their positive effects are discussed in Schoen & Hirsch (this MONTHLY, February 2003). Moreover, a revised 2nd edition is now being completed, with Course 1 just published. Thus, the Hill and Parker study does not address the published Core-Plus Mathematics curriculum as currently in use in schools.

The second and more serious problematic aspect of the paper is the way it misrepresents the high school curriculum experiences of students. Conclusions about the effectiveness of the Core-Plus Mathematics program are based upon the classification of six Michigan high schools as "only Core-Plus" or "supplemented Core-Plus." The main data analysis is restricted to just the former category of schools. However, four of the six schools are misclassified.

Two of the four schools in Table 1 that are classified as using "only Core-Plus" actually used a variety of mathematics curriculum materials with the graduating classes of 1998 and 1999. In fact, in 1998 only 40 of the 830 graduating seniors had experience with any Core-Plus Mathematics material. The school district reported this fact to Hill and Parker shortly after the study was first posted and then widely disseminated on the internet, but the classifications have remained unchanged.

Two schools are classified as "supplemented Core-Plus," due in part to the offering of a "precalculus" course. However, teachers at these two schools used Core-Plus Mathematics as their primary text in all four years, including for "precalculus." Thus, these schools are incorrectly classified as "supplemented Core-Plus," and they are not included in the main data analysis. Hill and Parker report separately on these schools, noting that, "the clear declines seen in the data from

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the four CP ["only Core-Plus"] schools are not evident here." Hill and Parker were informed in 2003 of the misclassification, but here too the classifications and inferences drawn have remained unchanged.

We agree that there is a need for research assessing effects of different high school mathematics experiences on students' progress in collegiate mathematics. However, we urge MONTHLY readers to look for more valid research based on *published* versions of curricula. A brief report of our project evaluators' start in this direction can be found at http://www.wmich.edu/cpmp/LongitudinalStudy1. html.

Here is a response from Richard Hill and Thomas Parker:

The letter from the Core-Plus group ("the Fey letter") begins by pointing to the new edition of the Core-Plus series, suggesting that shortcomings of earlier editions have been fixed. This may well be true; our study is simply a report of what happened at some schools as they implemented the Core-Plus program during the years described.

The contention that we misclassified some high schools is a result of not understanding our study design. This was *not a comparison study*. We did not group students according to whether or not they used Core-Plus materials, nor did we form "before-and-after" groups. Instead, we sought to compare each high school *with itself* over the years as the school phased-in the Core-Plus materials, looking for *trends over time* in the college mathematics performance of the graduates of each school who came to Michigan State. To minimize confounding variables, we focused on schools that were committed to moving to an all Core-Plus mathematics curriculum. This study design posits only that the number of students taking Core-Plus courses increased significantly over the years studied.

The data from each school begin with a baseline year in which Core-Plus materials were not used. From there, implementation proceeded at different rates in the various schools we studied. At two of these schools the implementation was never completed because, following parents' outcry against the program, alternative options were introduced in the 1999–2000 school year.

Thus we did not classify any school as using "*only* Core-Plus," as the letter mistakenly asserts. We explained on page 909 that our "Core-Plus group" consists of students from schools "that *were implementing* a system of offering only Core-Plus mathematics." This criterion was met by the two schools that the Fey letter contends were misclassified, and the number they claim for the year 1998 is compatible with our description (taken from an August, 1999 article in the local newspaper) that "some of the class of 1998, about half of the class of 1999, and most of the class of 2000 had taken Core-Plus courses" (page 915). For these two schools, we examined data for the years 1994–2000 and displayed it in Table 4 for readers to judge for themselves.

The decline in student performance seen in Table 4 is real. Perhaps the Fey letter is suggesting that this decline is so great and occurs so early in the implementation that it cannot be attributed to Core-Plus. Our paper cautions about ascribing such causality. In this case, we consulted a lead math teacher, a mid-level administrator, and the district Superintendent of Schools, and we searched a State of Michigan database, looking for other possible causes of the observed changes. We found none.

Our "supplemented Core-Plus" schools are not misclassified. As explained on page 917, these schools augmented the program in ways the other four schools

did not. One listed on its web site that it had a separate, algebraically-demanding precalculus course, so did not qualify for the "Core-Plus group" as defined on page 909. The situation was less clear for the other school. One of us (Hill) visited the school, talked to faculty members about how they implemented Core-Plus, and listened carefully to those who suggested their data ought to be included on equal footing with the four schools in the main part of our study. We still concluded that this school did not meet our criterion for the "Core-Plus" group.

Nevertheless, to be thorough, we combined the student data from this school with the data from Table 1, analyzed the combined data, and reported the results in footnote 4 on page 912. The effect of reclassifying this school is minimal and does not alter the statistical conclusions at all. This is laid out clearly, without deception, in our paper.

The Fey letter concludes by urging readers of this MONTHLY to "look for more valid research [than ours] based on *published* versions of curricula." We agree that such research should be an ongoing enterprise, but we disagree that pilot programs that are applied to whole-school populations are *ipso facto* unworthy of study. Finally, when examining research like the one-year study recommended by the authors of the Core-Plus program, readers should be alert to the conflicts that may arise when the same group of people serves as grant recipients, writers, advertisers, and evaluators for a particular mathematics program.

Enrique Arrondo (Universidad Complutense, Madrid) submitted the following comment about his February 2006 note:

> As Prof. Marco Manetti has kindly told me, the proof in my note "Another Elementary Proof of the Nullstellensatz," published in the February 2006 issue of this MONTHLY, was first found by him independently, and published in his notes "Corso Introduttivo alla Geometria Algebrica" (Italian) [Introduction to Algebraic Geometry], Appunti dei Corsi Tenuti da Docenti della Scuola [Notes of Courses Given by Teachers at the School] Scuola Normale Superiore, Pisa, 1998, 252 pp., reviewed in Mathematical Reviews: MR1738541 (2001f:14002).

Herman Roelants (University of Leuven, Belgium) sent in the following remarks:

> R. E. Hartwig's "Note on a Linear Difference Equation" (March 2006, pp. 250–256) certainly was interesting, but there are some things to be put straight:
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> 1. "Recurring series" is an old and venerable subject and history should not be completely forgotten. Hartwig's "user-friendly" solution is in essence a rediscovery of a result that goes back to H. Siebeck, "Die Recurrenten Reihen vom Standpunkte der Zahlentheorie aus Betrachtet," *J. Reine Angew. Math.* **33** (1846) 71–77. Therein the "user-friendly" solution is given for Hartwig's $u_{n+1} = pu_n + qu_{n-1}$ ($u_0 = 0$, $u_1 = 1$). Regrettably, an error in the last term is also repeated in L. E. Dickson's mention of this result (*History of the Theory of Numbers*, vol. I, 1952, p. 394). The same result can be found in several publications of E. Lucas who gave also a more systematic treatment in Chapter XVIII of *Théorie des Nombres* (1891, reprint Paris, 1961). It is interesting that Siebeck discusses also explicitly Hartwig's Case 1 and Case 2.
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> 2. There is no real need for the "master" equation. Indeed it is easily seen that if $y_{n+1} = py_n + qy_{n-1}$ ($y_0 = b$, $y_1 = a$) then $y_n = au_n + bqu_{n-1}$, a relation