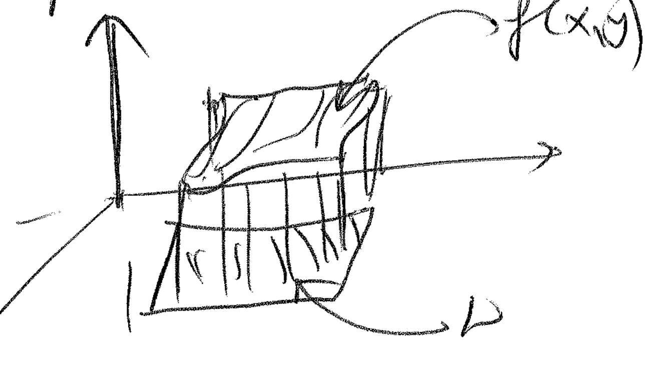
LECTURE 14 [section 15.5]

Recall,

$\cancel{\iint_D}$

1. $\iint_D f\,dA$ – is a volume of a subgraph.

what is subgraph?



2. $\iint_D 1\,dA$ – is the area of D

   (or it is a volume of subgraph of a function $f(x,y)=1$)

3. We learned two different ways of computing $\iint_D f\,dA$

(3.1) Cartessian coordironete system

$$\iint_D f\,dA = \cancel{\int_0^0\int_0^0 f(x,y)\,dx\,dy}$$

$$= \int_a^b \int_{g_1(y)}^{g_2(y)} f(x,y)\,dx\,dy = \int_c^d \int_{h_1(x)}^{h_2(x)} f(x,y)\,dy\,dx$$

and

(3.2) Poral coordinate system

$$\iint_D f\,dA = \int_{\theta_1}^{\theta_2} \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos\theta, r\sin\theta)\,\underline{r\,dr\,d\theta}$$

Remark: it is volume as well and if $f=1$ it is an area

How to find the limits?

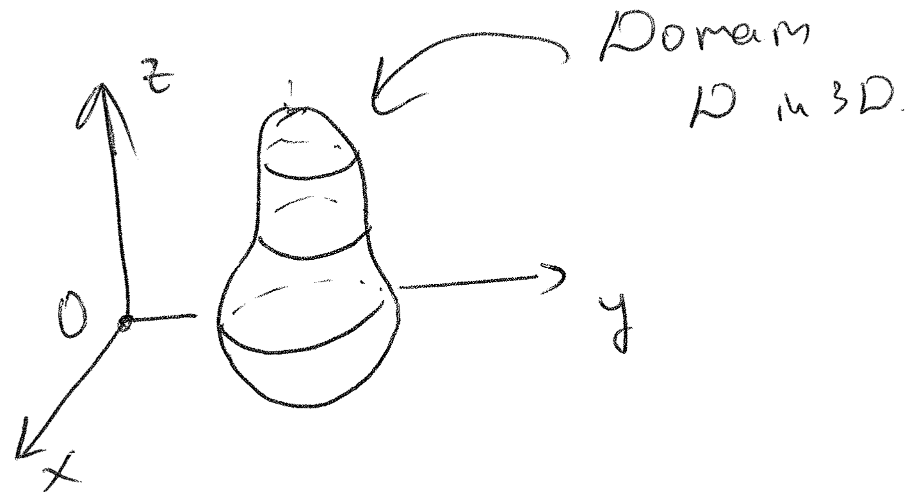
You shouldave solved 2D homework.

Tripple integrals

Let $f(x,y,z)$ - function of 3 variables.

Now we should integrate overl the sets in 3D (not over rectangles and domains in 2D)

Example of such domains:



How to define an integral of $f(x,y,z)$?

Since it has 3 variables we write triple integral, so

$$\int_D f\, dA = \iiint_D f \boxed{d\,\text{smth}}$$

There are several ways of computing this triple integral. Today we are going to learns computation in cartesian coordinate system. So

$$\iiint_D f\, d\text{smth} = \iiint f\, dx\,dy\,dz$$

limits

and you can change the order

$$\iiint f(x,y,z)\,dx\,dy\,dz$$

$$\iiint f(x,y,z)\,dy\,dx\,dz$$

$$\iiint f(x,y,z)\,dy\,dz\,dx$$

$$\iiint f(x,y,z)\,dx\,dz\,dy$$

$$\iiint f(x,y,z)\,dz\,dx\,dy$$

$$\iiint f(x,y,z)\,dz\,dy\,dz$$

Wow! 6 different ways! And each of them has different limits. Let's learn how to do in the order ~~dx dy dz~~

$dz\,dx\,dy$

So what are the limits

$$\iiint f(x,y,z)\,dz\,dx\,dy$$

before we start computing lets make the following remarks:

~~$\iiint_D (d_{3\text{-}with})$ ... is the volume of D.~~

~~Example~~ $\iiint_D dA$ - is the volume of D.

Recall: ① $\int_{\text{interval}} dx$ = length of interval

② $\iint_{2D} dA$ = area of 2D

③ $\iiint_{3D} dA$ = Volume of 3D

(1.1) $\int_{\text{interval}} \cancel{f(x)}\, dx$ = area of a subgraph f over the interval

(2.1) $\iint_{2D} \cancel{f(x,y)}\, dA$ = Volume of a subgraph of f over the 2D domain.

(3.1) $\iiint_{3D} f(x,y,z)\, dA$ = don't need to know

Average values

① $\frac{1}{\text{length(interval)}} \int_{\text{interval}} f\, dx$ = average value of f (1 dim)

② $\frac{1}{\text{area(2D domain)}} \iint_{\text{2D domain}} f\, dA$ = average value of f (2 Dim)

③ $\frac{1}{\text{Volume(3D domain)}} \iiint_{\text{3D domain}} f\, dA$ = average value of f (3 Dim)

Let's return:

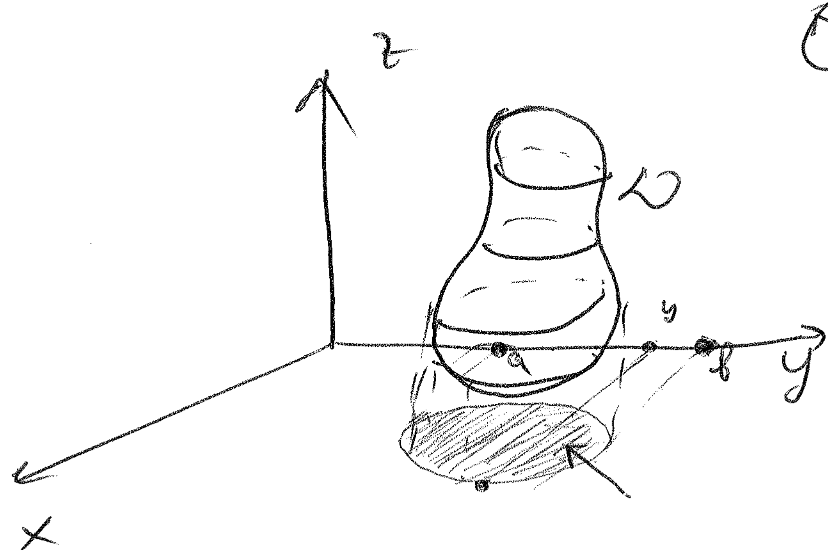
How to compute

$$\iiint_D f(x,y,z)\, dz\, \underbrace{dx\, dy}$$

project onto xy plane

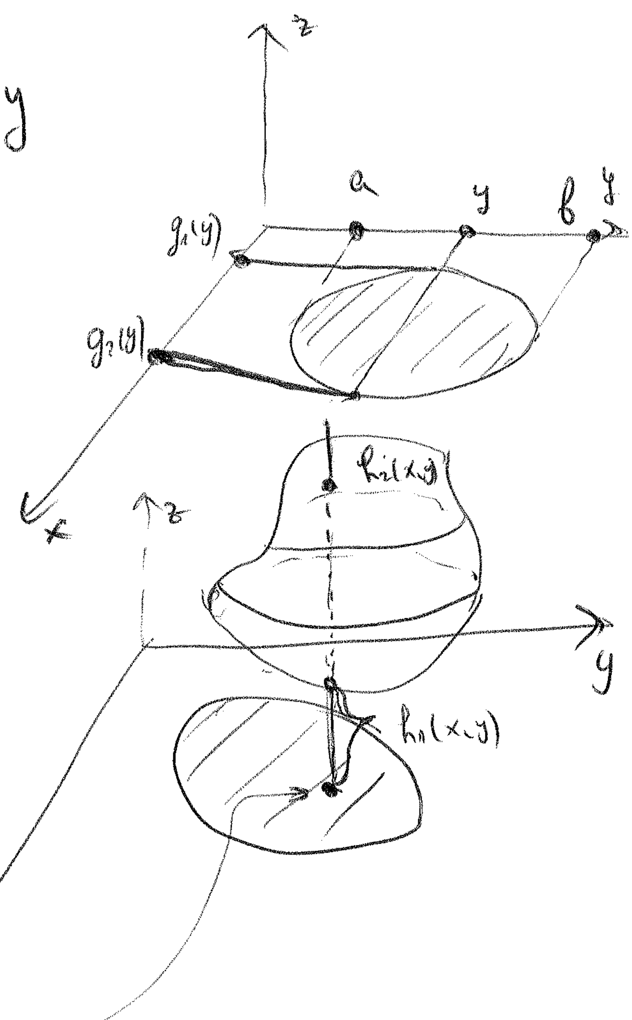
1st step: If order is this

(i.e. las two variables)



Then write limits as for double integrals

$$\int_a^b \int_{g_1(y)}^{g_2(y)} \int f(x,y,z) \qquad dx\,dy$$



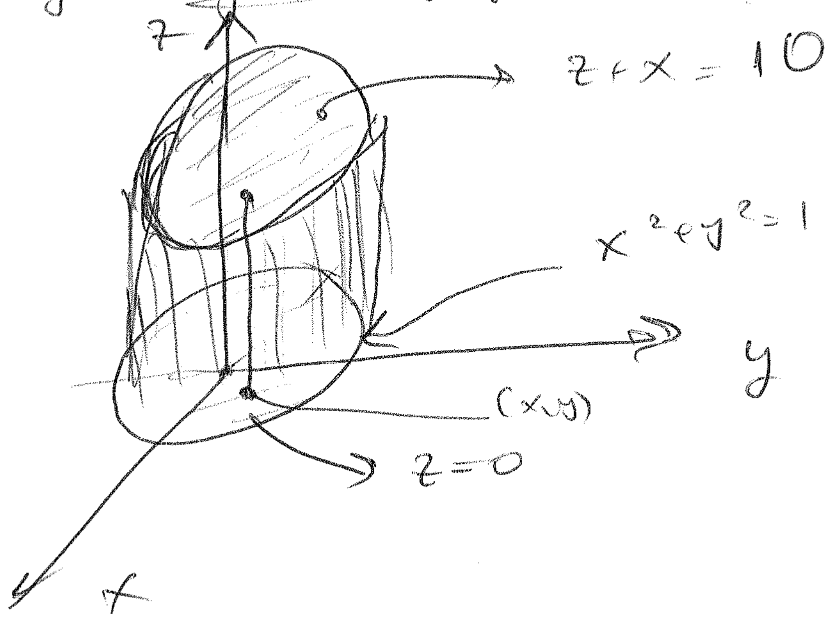
what about z?

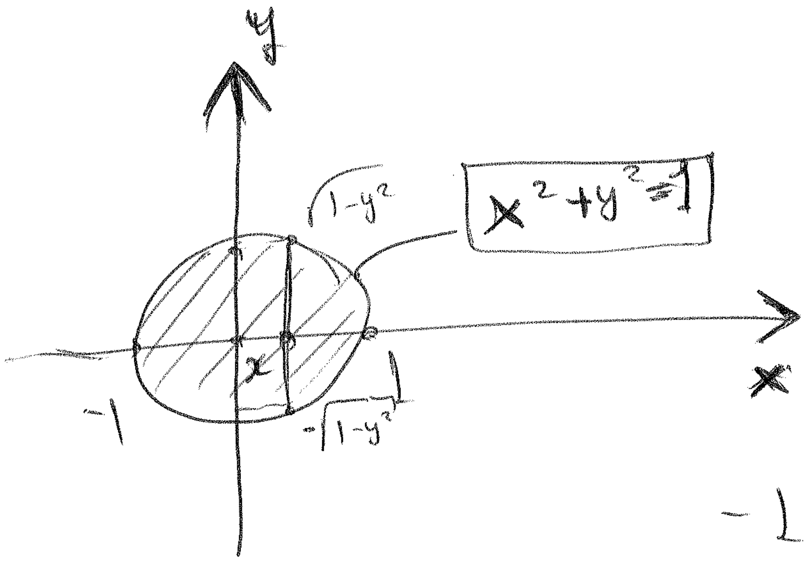
for each point (x,y) in this projection you need to draw a vertical line parralel to z axis and see when does it first enters and last enter domain 3D

$$\int_a^b \int_{g_1(y)}^{g_2(y)} \int_{h_1(x,y)}^{h_2(x,y)} f(x,y,z)\,dz\,dx\,dy$$

<u>Examples:</u> see all examples in the book

Find the following ~~integrals~~ Volume:





$-1 \le x \le 1$ for each given x

$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$ for each given (x,y)

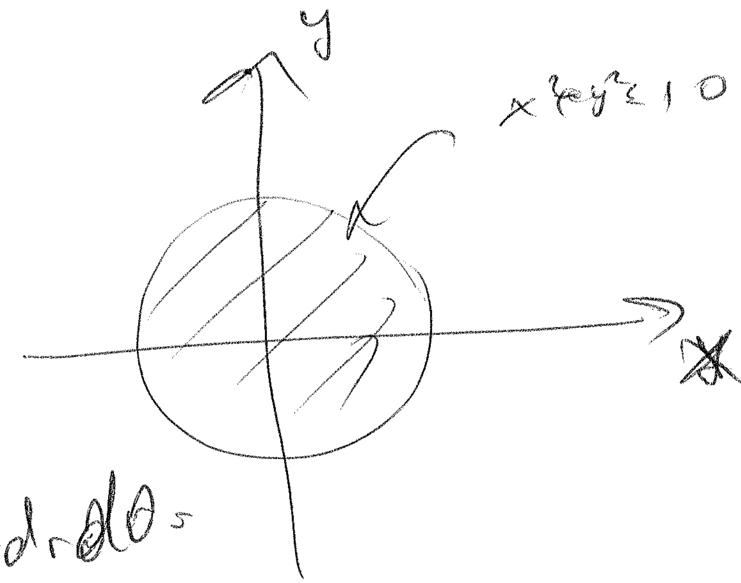
$0 \le z \le 10-x$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{10-x} dz\,dy\,dx =$$

$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (10-x)\,dy\,dx = \int_{-1}^{1} 2(10-x)\sqrt{1-x^2}\,dx$$

but that integral is hard to compute ☹. So we will do the following

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (10-x)\,dy\,dx = \text{lets go to polar coordinates}$$



$$\text{So } = \int_0^{2\pi}\int_0^{1} (10 - r\cos\theta)\, r\,dr\,d\theta =$$

$$= \int_0^{2\pi}\int_0^{1} 10r\,dr\,d\theta - \int_0^{2\pi}\int_0^{1} r^2\cos\theta\,dr\,d\theta =$$

$$= 2\pi \cdot \frac{10r^2}{2}\Big|_{r=0}^{r=1} - 0 = \boxed{10\pi}$$

I am not considering examples in this lecture notes

So ~~if~~ you are reading this right now

You should (or must) review all examples ~~on~~ covered in the book. And you should also solve homeworks