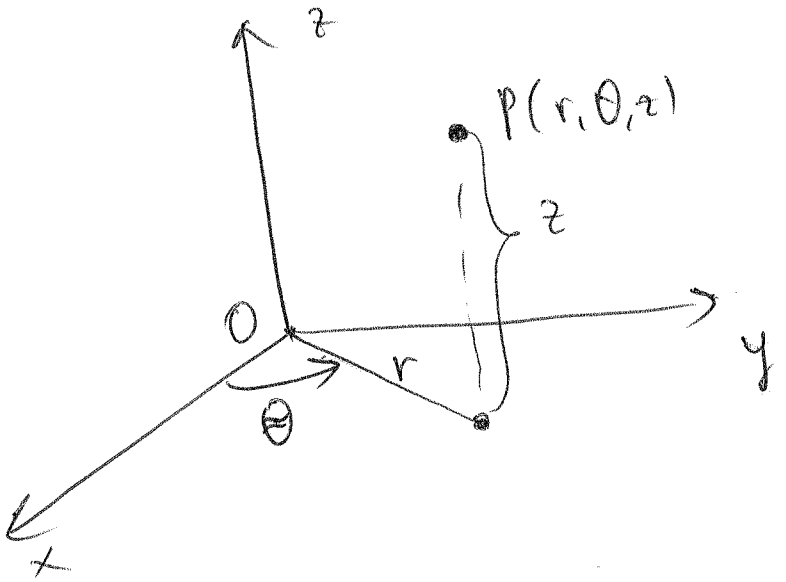
LECTURE 15

Triple integrals in Cylindrical and Spherical coordinate system

Section 15.7

Instead of (x, y, z) you use (r, θ, z)



z - is a hight, r is distance between Origin and Projection of a point P onto XY plane and θ is an angle between X axis and the projected point (counterclockwise)

How do we integrate in cylindrical coordinate system?

The function might be given as $f(x,y,z)$ or $f(r,\theta,z)$

Example: $f(x,y,z) = x^2+y^2+z$

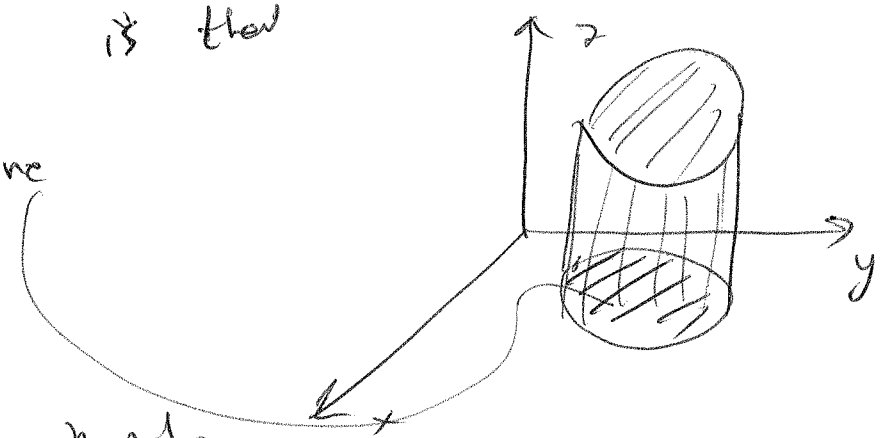
Example: $f(r,\theta,z) = r^2 + z\theta$

$$\iiint dz\, r\, dr\, d\theta$$

And pay attention that there is a number.

~~Example: Let $r = 2\cos\theta$~~

So what happens is the



projection onto xy plane you integrate in polar coordinate system

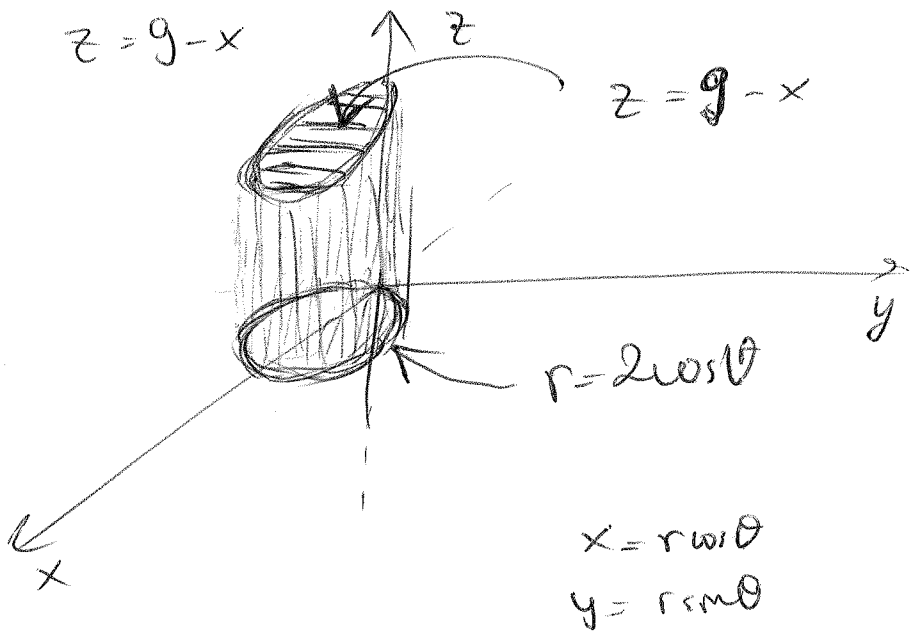
(so you find the limits in polar coordinate)

Example.

~~this~~ this is a disk

$r = 2\cos\theta$ in xy plane (cylinder)

and to the top $z = 9 - x$



$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$0 \le r \le 2\cos\theta$$

$$0 \le z \le 9 - r\cos\theta$$

$x = r\cos\theta$

$y = r\sin\theta$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{2\cos\theta} \int_{0}^{r\cos\theta} dz\, r\, dr\, d\theta$$

So first you write dz and find its limits. Then you write this r and find its limits. Then you write this dr and find its limits.

Done!

Spherical coordinate system

(4)

function might

instead of $\longrightarrow$ (ρ, φ, θ)

we

have

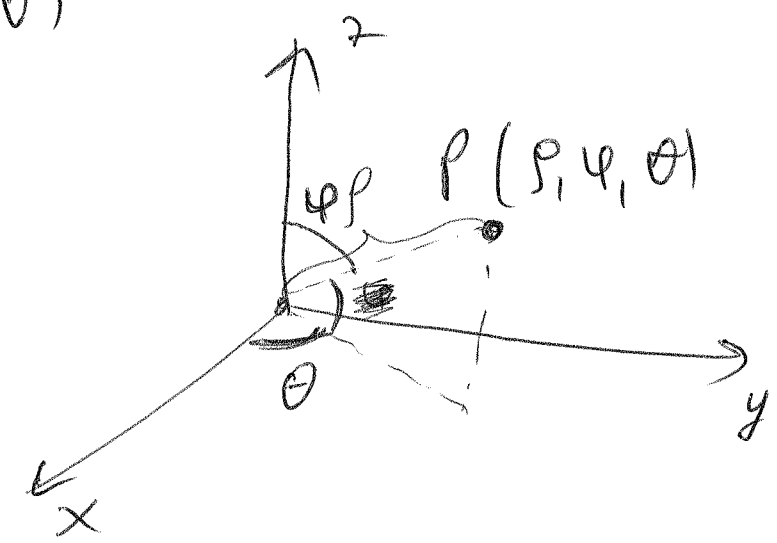
instead of (x, y, z)

we have (ρ, φ, θ)

$\rho \geq 0$

$0 \leq \varphi \leq \pi$

$0 \leq \theta \leq 2\pi$



ρ - is a distance between point P and the origin. θ is an angle (like in polar coordinates) and φ is an angle between z axis and $\overrightarrow{OP}$

$$\iiint f(\cdot) \; \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

Remember

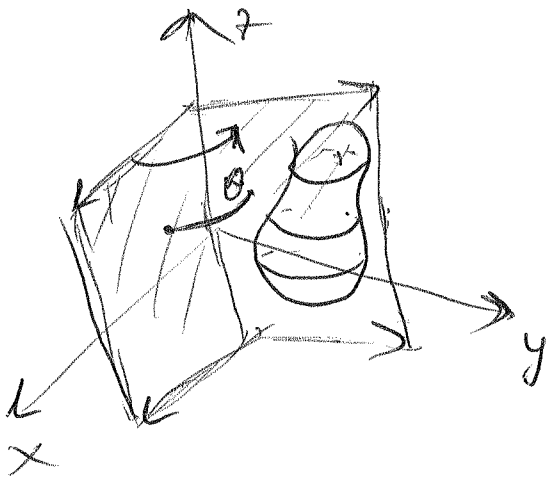
$$x = \rho \cos\theta \sin\varphi$$

$$y = \rho \sin\theta \sin\varphi$$

$$z = \rho \cos\varphi$$

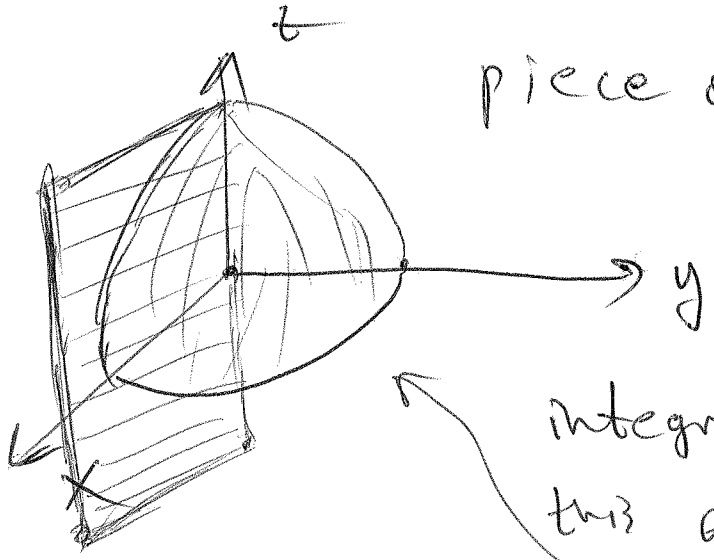
So if you integrate a function $f(x,y,z)$ then it becomes $f(\rho\cos\theta\sin\varphi, \rho\sin\theta\sin\varphi, \rho\cos\varphi)$

1) For finding the values of θ you rotate a plane (arrow ~~thing~~ like plane



First moment when does it touch and the last moment.

(Angle you measure between xz plane and this rotated plane (counterclockwise))



piece of ball $\rho \leq 2$
of radius $= 2$

integrate $f(x,y,z)$ over this domain

$$\iiint \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

Find the limits of θ

$$0 \leq \theta \leq \frac{\pi}{2}$$

(angle between a door and x axis
first time it intersects and the
last time

$$0 \leq \varphi \leq \frac{\pi}{2}$$

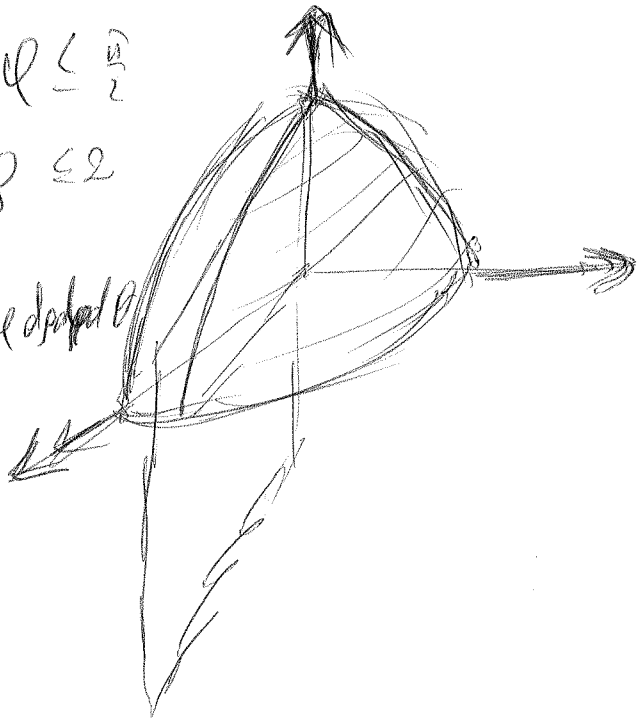
$$0 \leq \rho \leq 2$$

$$0 \leq \theta \leq \frac{\pi}{2}$$

$$0 \leq \varphi \leq \frac{\pi}{2}$$

$$0 \leq \rho \leq 2$$

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^{2} f(\rho\cos\theta\sin\varphi, \rho\sin\theta\sin\varphi, \rho\cos\varphi)\, \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$



Please Review examples
in the book