

INFORMAL LECTURE NOTES

MTH 234.

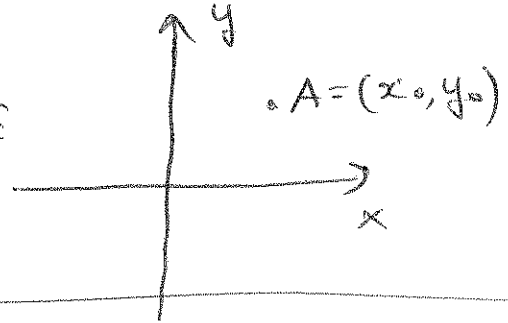
①

LECTURE 1.

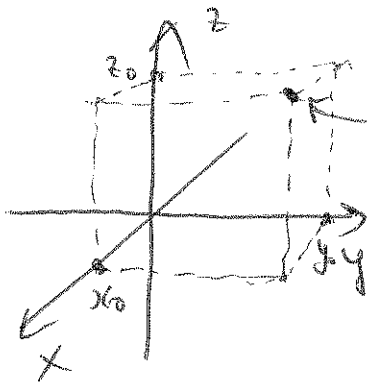
Section 12.1

2D - COORDINATE SYSTEM

Example: $y = 2x + 1$ IS A LINE



3D - COORDINATE SYSTEM.

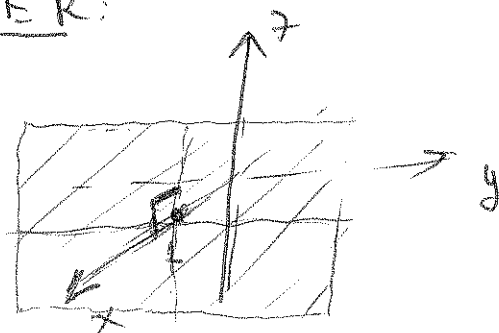


$A = (x_0, y_0, z_0)$

POINTS HAVE 3 COORDINATES

Q: DESCRIBE SET OF POINTS IN 3D (x, y, z) such that $x = 1$.

ANSWER:

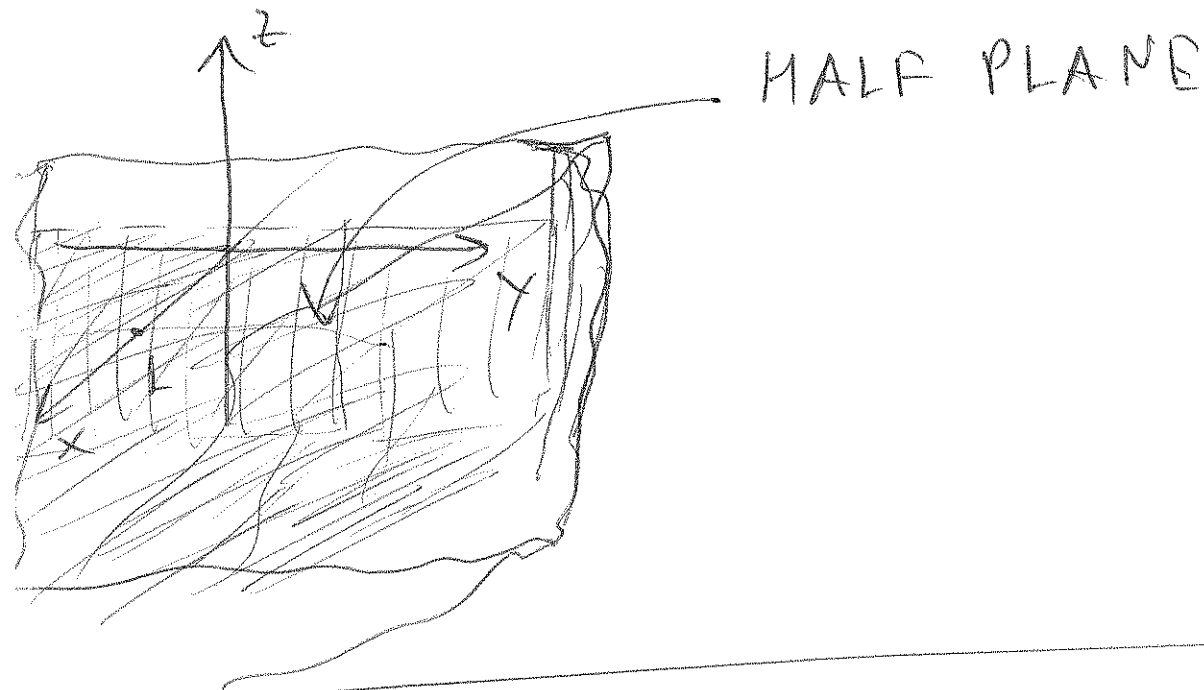


IT IS A PLANE PERPENDICULAR TO THE X-AXIS AT POINT $x = 1$.

Q: ~~x > 1~~ Describe $x \geq 1$.

②

ANSWER: IT IS A HALF PLANE



REMEMBER: ~~SOM~~ SOMETHING = 0

Gives Boundary (Surface).

And SOMETHING ≥ 0 is one side
of this boundary and SOMETHING ≤ 0

Is another side of this boundary

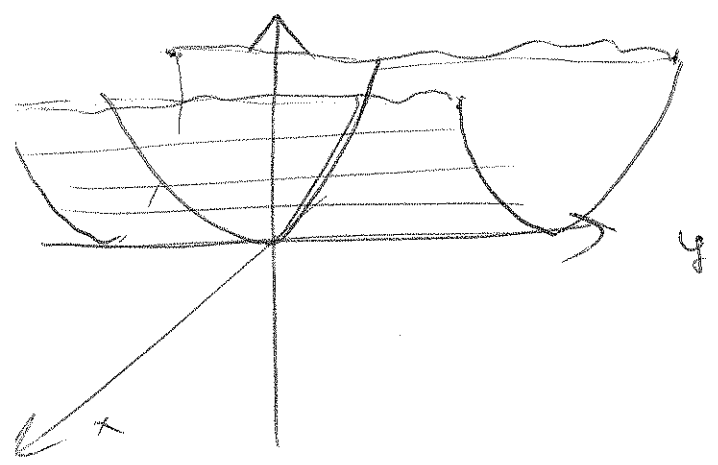
Q. Describe

3

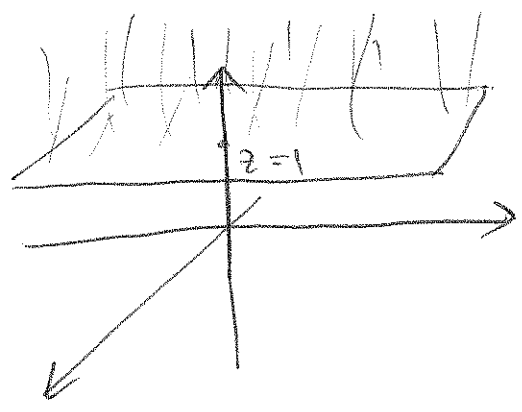
$$\begin{cases} y = x^2 \\ z \geq 1 \end{cases}$$

Answer:

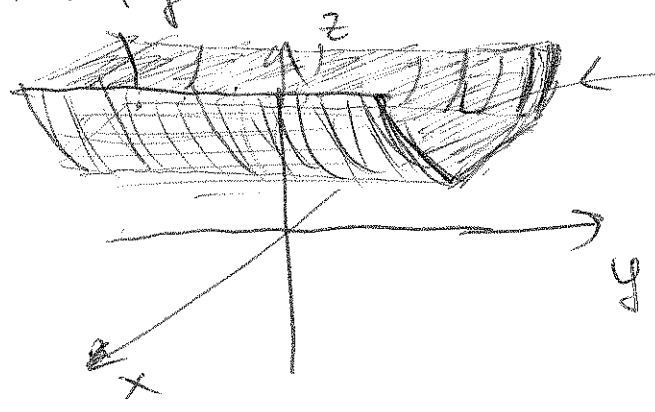
First we draw $y = x^2$ in 3D.



AFTER THAT WE DRAW $z \geq 1$



Finally we take intersection:



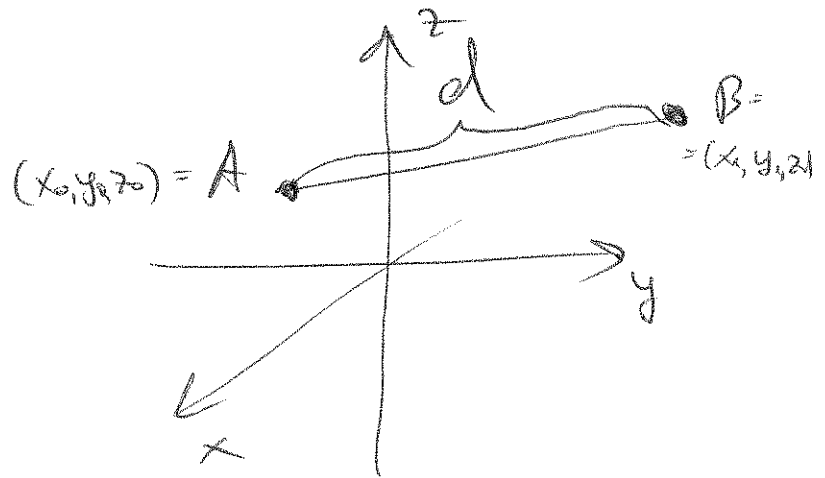
$$\begin{cases} y \geq x^2 \\ z \geq 1 \end{cases}$$

DISTANCE BETWEEN POINTS

(4)

Let $A = (x_0, y_0, z_0)$

$B = (x_1, y_1, z_1)$



What is the value of $d = ?$

$$d = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

PROOF: The Pythagorean theorem.

EQUATION OF A SPHERE OF RADIUS R

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$$

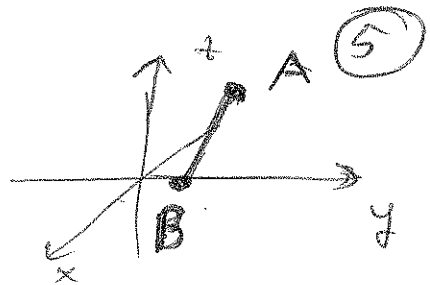
where (x_0, y_0, z_0) is a center of a sphere and R is a radius.

GOOD TO KNOW:

This equation tells you that distance between the points (x, y, z) and (x_0, y_0, z_0) is constant $= R$.

Example:

$$A = (1, 2, 3), \quad B = (0, 1, 0)$$



$$\begin{aligned} \text{Distance} &= \sqrt{1^2 + (2-1)^2 + 3^2} = \\ &= \sqrt{1 + 1 + 9} = \sqrt{11} \end{aligned}$$

Example:

Describe the set of points (x, y, z) such that

$$x^2 + 2x + y^2 + z^2 - 2z = 10$$

Solution:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\begin{aligned} 0 &= (x^2 + 2x) + y^2 + (z^2 - 2z) = \\ &= (x^2 + 2x + 1) - 1 + y^2 + (z^2 - 2z + 1) - 1 = \\ &= (x+1)^2 + y^2 + (z-1)^2 - 2 \end{aligned}$$

Hence:

$$12 = (x+1)^2 + y^2 + (z-1)^2$$

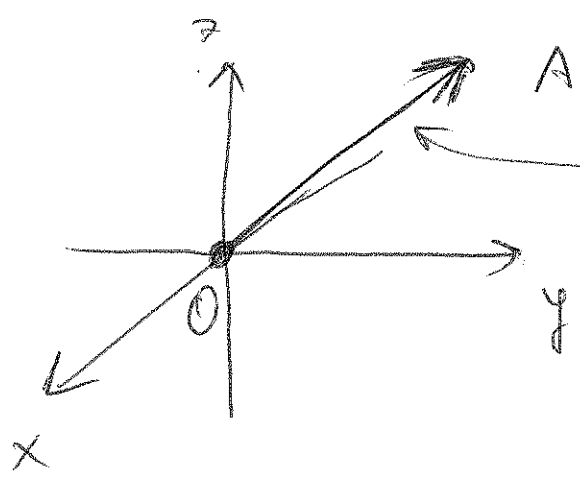
$$\text{radius} = \sqrt{12}$$

$$\text{center} = (-1, 0, 1)$$

Section 12.2

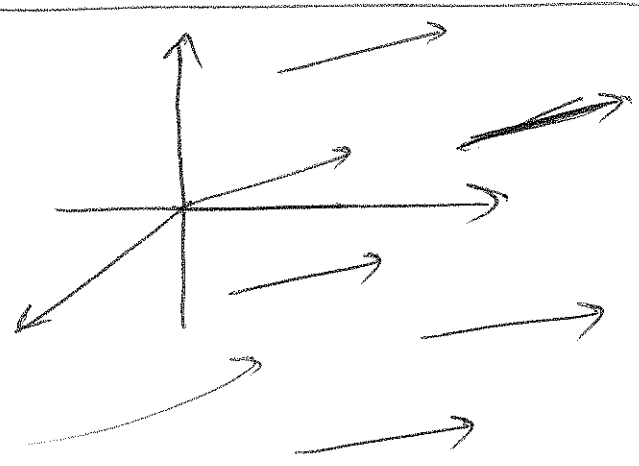
6

IN 3D WE HAVE A POINT
 $A = (x_0, y_0, z_0)$. WHAT IS VECTOR?



vector $\vec{OA} = \langle x_0, y_0, z_0 \rangle$
different brackets

WHEN WE SAY VECTOR \vec{OA} or \vec{v}
we care about length of OA and
direction.



These are
one and the same
vectors because
they have the same
length and direction.

4) IF $\vec{v} = \langle v_1, v_2, v_3 \rangle$ Properties

(4)

Then $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

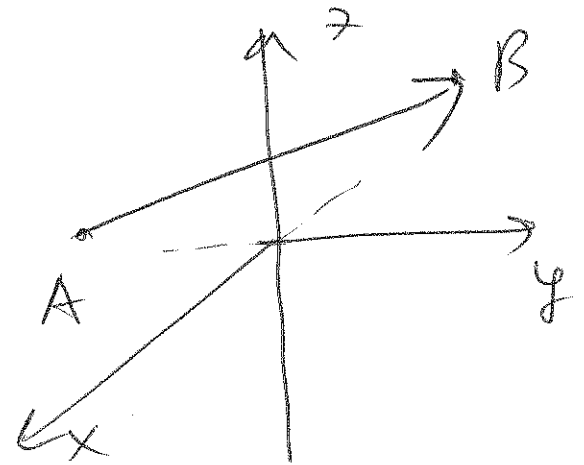
length amplitude magnitude

2) IF $A = (x_0, y_0, z_0)$

$B = (x_1, y_1, z_1)$

$$\vec{AB} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

$$= \vec{OB} - \vec{OA} \quad \text{Therefore} \quad \vec{AB} = \vec{AO} + \vec{OB}$$



$$3) \vec{AB} = -\vec{BA}$$

Example: $A = (0, 1, 2)$

$B = (3, 4, 5)$

$\vec{AB} = ?$

Answer: $\vec{AB} = \langle 3-0, 4-1, 5-2 \rangle =$

$$= \langle 3, 3, 3 \rangle$$

$$1) \text{ IF } \vec{v} = \langle v_1, v_2, v_3 \rangle$$

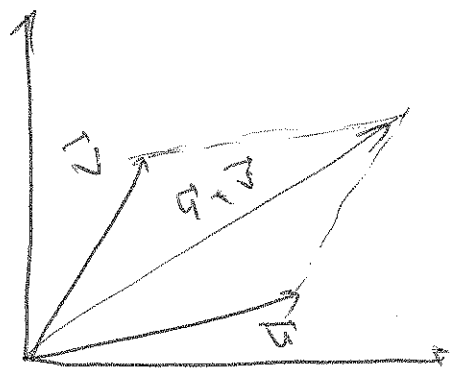
8

$$\text{Then } \vec{u} = \langle u_1, u_2, u_3 \rangle$$

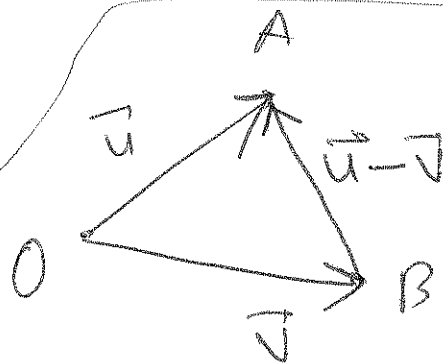
$$\vec{v} + \vec{u} = \langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle$$

5.) IF $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and λ is a number (scalar) then

$$\lambda \cdot \vec{v} = \langle \lambda v_1, \lambda v_2, \lambda v_3 \rangle$$



Parallelogram law



IF

$$\vec{OA} = \vec{u}$$

$$\vec{OB} = \vec{v}$$

then

$$\begin{aligned} \vec{BA} &= \vec{BO} + \vec{OA} = -\vec{OB} + \vec{OA} = \\ &= -\vec{v} + \vec{u} \end{aligned}$$

IF k is a number then

(9)

Then

$$|k \cdot \vec{u}| = |k| \cdot |u|$$

PROOF: BY DEFINITION.

Example:

$$\vec{u} = \langle 1, 2, 3 \rangle, \quad \vec{v} = \langle 1, 1, 1 \rangle$$

Find $\frac{1}{2} \vec{u} - 3 \vec{v} = ?$

Answer: $\frac{1}{2} \vec{u} - 3 \vec{v} = \frac{1}{2} \langle 1, 2, 3 \rangle -$

$$\begin{aligned} - 3 \cdot \langle 1, 1, 1 \rangle &= \left\langle \frac{1}{2}, 1, \frac{3}{2} \right\rangle - \langle 3, 3, 3 \rangle = \\ &= \left\langle -\frac{5}{2}, -2, -\frac{3}{2} \right\rangle \end{aligned}$$

More properties:

~~u~~

IF \vec{v} is a vector

(10)

then $\frac{\vec{v}}{|\vec{v}|}$ - is called direction.

The unit vector is a vector such that its length is 1.

$$\underline{|\vec{v}| = 1}$$

Proposition: IF \vec{u} is a vector

then $\frac{\vec{u}}{|\vec{u}|}$ - is a unit vector

PROOF: INDEED:

$$\left| \frac{\vec{u}}{|\vec{u}|} \right| = \frac{1}{|\vec{u}|} \cdot |\vec{u}| = 1.$$

Example: $\vec{u} = \left\langle \frac{3}{5}, \frac{4}{5}, 0 \right\rangle$ - is a

Indeed

$$|\vec{u}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 + 0^2} = \sqrt{\frac{9+16}{25}} = 1.$$

unit vector