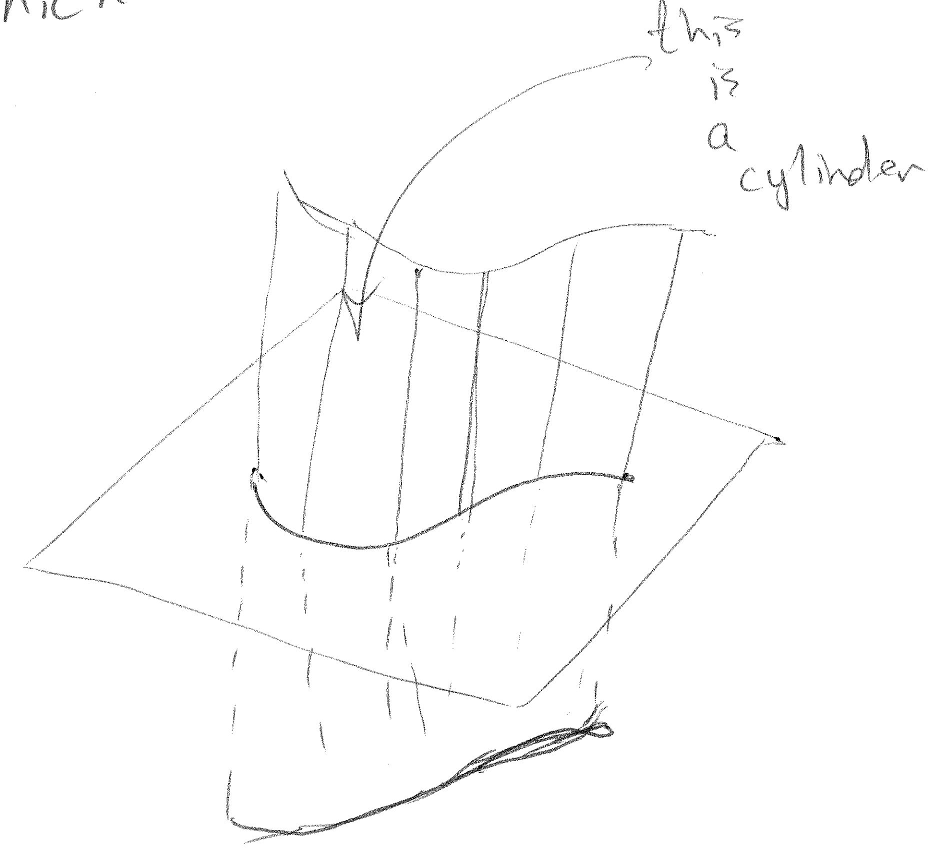
Lecture 4:

Cylinder and Quadric Surfaces.

What is cylinder?

Take any plane curve (curve which lies in some plane) and draw a straight lines which pass through that curve and which are perpendicular to that plane

Example:



Remark:

1) line is a cylinder.

2) Plane is a cylinder

3) Any equation in 3D in which we are missing at least one variable is a cylinder

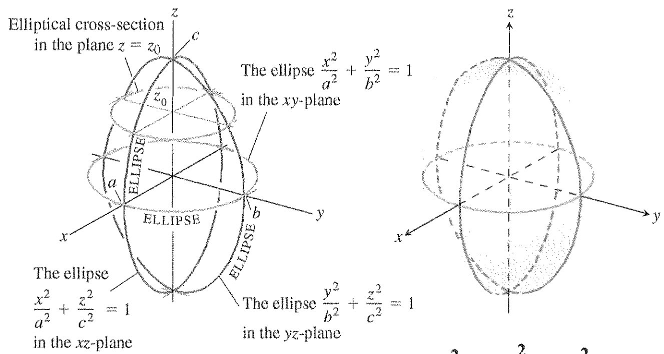
Example

$x = y^2$ in 3D is a cylinder

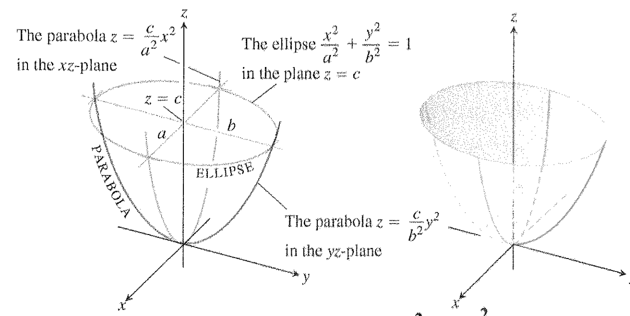
Cylinder

Picture

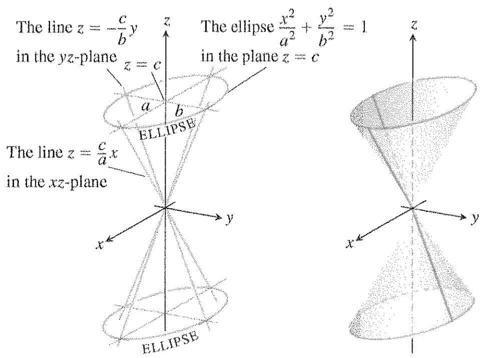
TABLE 12.1 Graphs of Quadric Surfaces



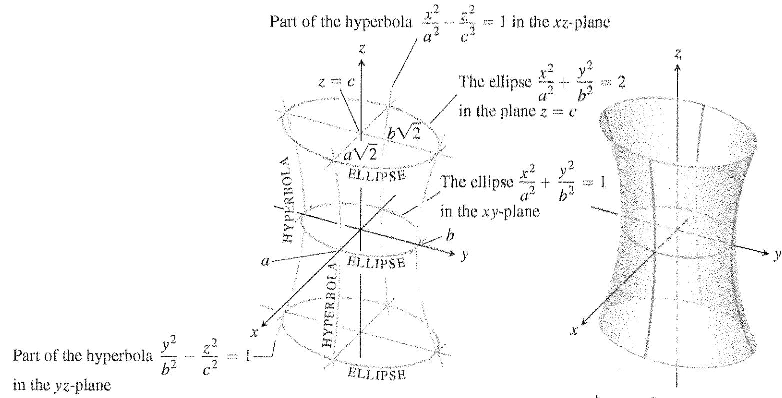
ELLIPSOID $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



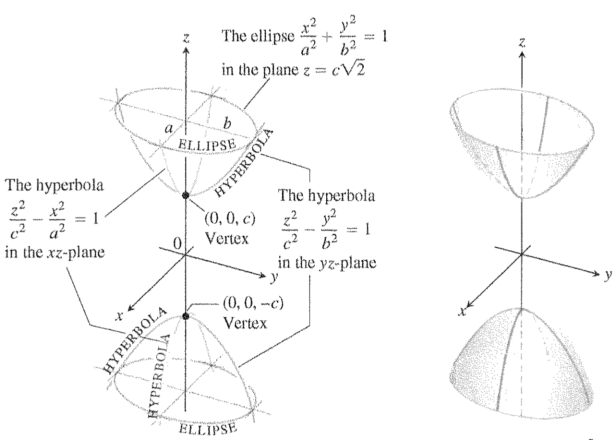
ELLIPTICAL PARABOLOID $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$



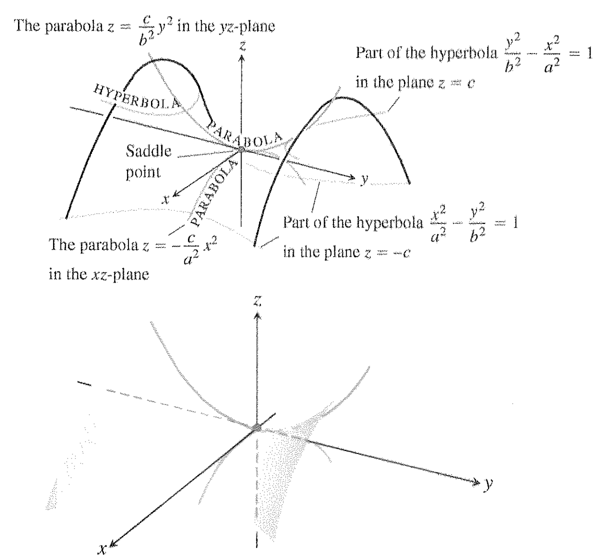
ELLIPTICAL CONE $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$



HYPERBOLOID OF ONE SHEET $$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



HYPERBOLOID OF TWO SHEETS $$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



HYPERBOLIC PARABOLOID $$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}, \quad c > 0$$

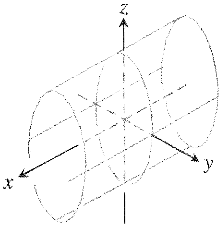
Exercises 12.6

Matching Equations with Surfaces

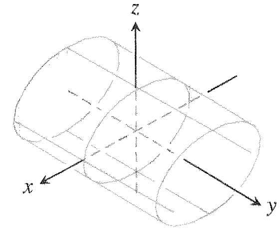
In Exercises 1–12, match the equation with the surface it defines. Also, identify each surface by type (paraboloid, ellipsoid, etc.) The surfaces are labeled (a)–(l).

1. $x^2 + y^2 + 4z^2 = 10$
2. $z^2 + 4y^2 - 4x^2 = 4$
3. $9y^2 + z^2 = 16$
4. $y^2 + z^2 = x^2$
5. $x = y^2 - z^2$
6. $x = -y^2 - z^2$
7. $x^2 + 2z^2 = 8$
8. $z^2 + x^2 - y^2 = 1$
9. $x = z^2 - y^2$
10. $z = -4x^2 - y^2$
11. $x^2 + 4z^2 = y^2$
12. $9x^2 + 4y^2 + 2z^2 = 36$

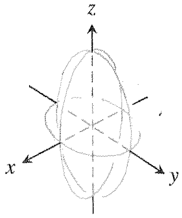
a.



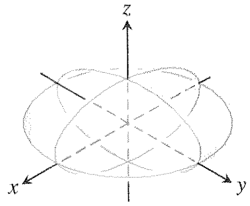
b.



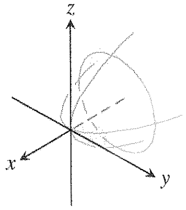
c.



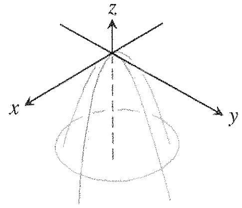
d.



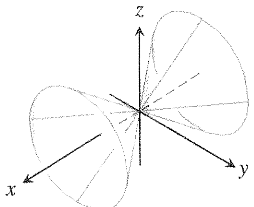
e.



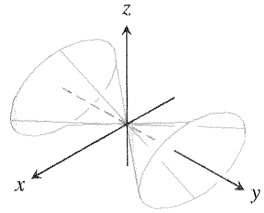
f.



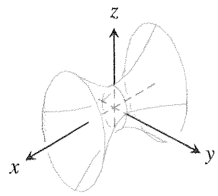
g.



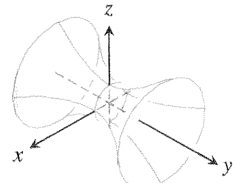
h.



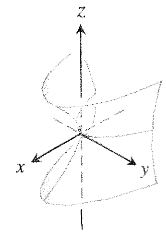
i.



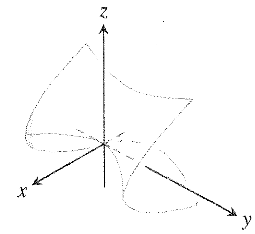
j.



k.



l.



Drawing

Sketch the surfaces in Exercises 13–44.

CYLINDERS

13. $x^2 + y^2 = 4$
14. $z = y^2 - 1$
15. $x^2 + 4z^2 = 16$
16. $4x^2 + y^2 = 36$

ELLIPSOIDS

17. $9x^2 + y^2 + z^2 = 9$
18. $4x^2 + 4y^2 + z^2 = 16$
19. $4x^2 + 9y^2 + 4z^2 = 36$
20. $9x^2 + 4y^2 + 36z^2 = 36$

PARABOLOIDS AND CONES

21. $z = x^2 + 4y^2$
22. $z = 8 - x^2 - y^2$
23. $x = 4 - 4y^2 - z^2$
24. $y = 1 - x^2 - z^2$
25. $x^2 + y^2 = z^2$
26. $4x^2 + 9z^2 = 9y^2$

HYPERBOLOIDS

27. $x^2 + y^2 - z^2 = 1$
28. $y^2 + z^2 - x^2 = 1$
29. $z^2 - x^2 - y^2 = 1$
30. $(y^2/4) - (x^2/4) - z^2 = 1$

HYPERBOLIC PARABOLOIDS

31. $y^2 - x^2 = z$
32. $x^2 - y^2 = z$

ASSORTED

33. $z = 1 + y^2 - x^2$
34. $4x^2 + 4y^2 = z^2$
35. $y = -(x^2 + z^2)$
36. $16x^2 + 4y^2 = 1$
37. $x^2 + y^2 - z^2 = 4$
38. $x^2 + z^2 = y$
39. $x^2 + z^2 = 1$
40. $16y^2 + 9z^2 = 4x^2$
41. $z = -(x^2 + y^2)$
42. $y^2 - x^2 - z^2 = 1$
43. $4y^2 + z^2 - 4x^2 = 4$
44. $x^2 + y^2 = z$

Theory and Examples

45. **a.** Express the area A of the cross-section cut from the ellipsoid

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$$

by the plane $z = c$ as a function of c. (The area of an ellipse with semiaxes a and b is πab.)

b. Use slices perpendicular to the z-axis to find the volume of the ellipsoid in part (a).

c. Now find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Does your formula give the volume of a sphere of radius a if $a = b = c$?