LECTURE 6.

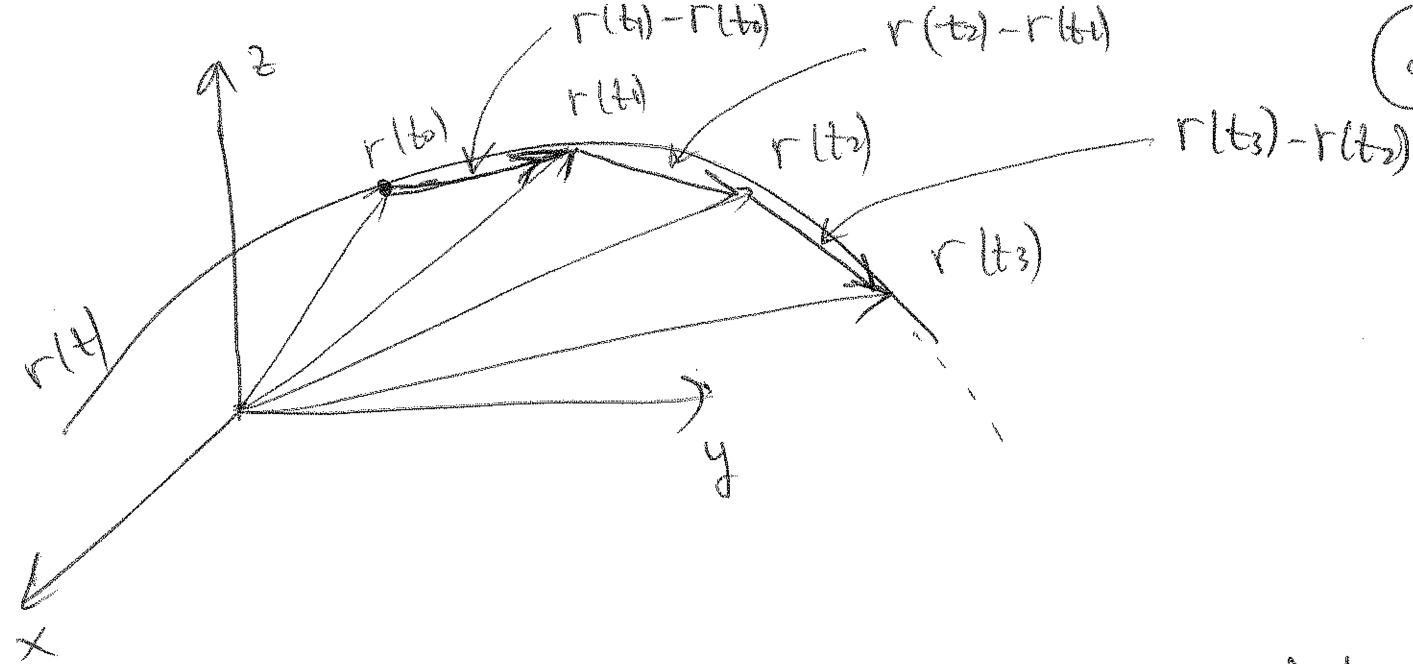
IF $r(t) = f(t)\cdot i + g(t)\cdot j + h(t)\cdot k$

Then the length of a curve from ~~time~~ t_0 ~~to~~ time t_1 is

$$L = \int_{t_0}^{t_1} |r'(t)|\,dt =$$

$$= \int_{t_0}^{t_1} \sqrt{(f')^2 + (g')^2 + (h')^2}\,dt$$

Explanation:



The length from point $r(t_0)$ to point $r(t_3)$ is <u>almost</u> the sum

$$|r(t_1)-r(t_0)| + |r(t_2)-r(t_1)| + |r(t_3)-r(t_2)| =$$

$$= \frac{|r(t_1)-r(t_0)|}{t_1-t_0}\cdot(t_1-t_0) + \frac{|r(t_2)-r(t_1)|}{t_2-t_1}(t_2-t_1) +$$

derivative

dt

$$+ \frac{|r(t_3)-r(t_2)|}{t_3-t_2} \approx \int_{t_0}^{t_3} |r'(t)| \cdot dt .$$

Example:

Let $r(t) = \cos t \cdot i + \sin t \cdot j + k$

Find the length of a curve on the interval ~~to 0~~, $0 \le t \le 1$

Solution:

$$\int_0^1 |r'(t)|\,dt = \cancel{\int_0^1 \langle \cos t,} \sin$$

$$= \int_0^1 |\langle -\sin t, \cos t, 0\rangle|\, dt =$$

$$= \int_0^1 \sqrt{\sin^2 t + \cos^2 t + 0^2}\, dt = \int_0^1 1\, dt = 1.$$