LECTURE 8

What is <u>Open set</u>?

The set without boundary is open

What is <u>closed set</u>? The set with boundary is closed

Section 14.3

<u>Partial derivatives</u>

When you differentiate a function of several variables you consider other variables as a constant.

<u>Example:</u>

$$f(x,y) = x^2 + y^2$$

$f_x(x,y) = 2x$

$f_y(x,y) = 2y$

Example:

$$f(x,y,z) = xyz + \cos xy$$

$$f'_x = yz - y\sin(xy)$$

Example:

$$f(x,y) = \ln\left(\frac{1}{x^2+y^2}\right)$$

$$f'_x = -\frac{2x}{x^2+y^2}$$

Implicit differentiation

$$z^2 + yx + yxz = 0$$

Find z_x (think ~~like~~ the function $z(x,y)$)

$$\cancel{2zz_x + 0 + y}$$

Solution: $2zz'_x + y + yz = 0$

$$z'_x = \frac{-y-yz}{2z}$$

Example

$\cos(xyz) = 0$ Find y_x - ?

~~Solution~~

$\cancel{\sin(xyz)}$

$\cos(xyz) = 0$

(Lets differentiate with respect to x)

$$-\sin(xyz)\cdot\left(yz + xy_x z\right) = 0$$

$$yz + xy_x z = 0 \qquad y_x = -\frac{yz}{xz} = -\frac{y}{x}$$

Show that the following function $f(x,y,z) = x^2 + y^2 + z^2$ satisfies th equation

$$f_{xx} + f_{yy} + f_{zz} = 6$$

Indeed,

$f_{xx} = 2$

$f_{yy} = 2$

$f_{zz} = 2$

Hence

$$f_{xx} + f_{yy} + f_{zz} = 6.$$

Example:

Let $f = x^2 + c \cdot y^2$.

Find c such that $f_{xx} + f_{yy} = 0$.

Solution:

$f_{xx} = 2$

$f_{yy} = 2c$

Hence $f_{xx} + f_{yy} = 2 + 2c =$

$= 2(1+c) = 0$

Hence $c = -1$