LECTURE 9

$\boxed{\text{Section 14.4}}$

The Chain Rule

If $w = f(x)$ and $x = g(t)$ then

$$\frac{dw}{dt} = \frac{dw}{dx}\frac{dx}{dt}$$

but what happens if we have a function of several variables

$w = f(x,y)$ and $x = g(t)$, $y = h(t)$

then

$$\boxed{\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}}$$

Example:

$w = x^2 + y^2, \quad x = \cos t, \quad y = t^2$

Find $\frac{dw}{dt}$

Solution:

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} =$$

$$= (2x)\cdot(-\sin t) + (2y)\cdot(2t) =$$

$$= (2\cdot\cos t)\cdot(-\sin t) + (2t^2)(2t) =$$

$$= -2\cos t\cdot\sin t + 4t^3$$

~~What if w(x, y, z) = ?~~

~~What if w(x, y, z) and~~

~~t = (0, 5)~~

Assume

$w = f(x, y, z)$ and

$x = g(s, t)$, $y = h(s, t)$, $z = r(s, t)$

Then

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial s}$$

and

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial t}$$

Example

$$w = x^2 + y^2 + z^2, \quad x = s + t, \quad y = s - t$$

$$z = s \cdot t$$

Find

$$\frac{dw}{ds} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} +$$

$$+ \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial s} =$$

$$= (2x) \cdot 1 + (2y) \cdot 1 + (2z) \cdot t =$$

$$= 2x + 2y + 2z \cdot t = 2(s+t) + 2(s-t)$$

$$+ 2s \cdot t \cdot t = 2s + 2t + 2s - 2t + 2st^2 =$$

$$= 4s + 2st^2$$