Homework 17 3.3

Problem 5.

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$$5xy'' + 5y' - 2y = 0$$

$$y = \sum_{n=0}^{\infty} a_n (x - x_0)^{n+r} = \sum_{n=0}^{\infty} a_n x^{n+r}, \text{ since } x_0 = 0$$

$$y' = \sum_{n=0}^{\infty} a_n \cdot (n+r) x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} a_n \cdot (n+r)(n+r-1) x^{n+r-2}$$

Let's plug

$$5x \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-2} + 5 \sum_{n=0}^{\infty} a_n \cdot (n+r) x^{n+r-1} - 2 \sum_{n=0}^{\infty} a_n x^{n+r} = 0$$

Hence:

$$5 \sum_{n=0}^{\infty} a_n (n+r)(n+r-1) x^{n+r-1} + 5 \sum_{n=0}^{\infty} a_n \cdot (n+r) x^{n+r-1} - 2 \sum_{n=0}^{\infty} a_n x^{n+r} = 0$$

Coefficients in front of the same powers of x are equal to 0.

So the lowest power is (when $n=0$) $r-1$ and the total coefficient is $5a_0 r \cdot (r-1) + 5a_0 \cdot r = 0$, $p(r) = 5a_0 r(r-1) + 5a_0 \cdot r$

Condition $p(1) = 5$ implies that $5 \cdot a_0 = 5$, therefore $\boxed{a_0 = 1}$. So we found a_0. Hence

$$p(r) = 5r(r-1) + 5r = 5r^2$$

$P(r)=0$ if and only if $r=0$. So $r_+=r_-=0$.

$$5\sum_{n=0}^{\infty} a_n(n+r)(n+r-1)X^{n+r-1}+5\sum_{n=0}^{\infty} a_n\cdot(n+r)X^{n+r-1}-$$
$$-2\sum_{n=0}^{\infty} a_n X^{n+r}=0$$

Let's write total coefficient in front of X^{n+r}

$$5a_{n+1}\cdot(n+1+r)(n+1+r-1)+5a_{n+1}\cdot(n+1+r)-2a_n=0$$

$$a_{n+1}\left[5(n+1+r)(n+r)+5(n+1+r)\right]-2a_n=0$$

$$a_{n+1}=\frac{2}{5(n+1+r)(n+r)+5(n+1+r)}a_n.$$

and for the next question you plug the largest root

In our case $r=0$.

$$a_{n+1}=\frac{2a_n}{5(n+1)\cdot n+5(n+1)}=\frac{2a_n}{5(n+1)^2}$$

In the last question $a_0=1$

$$y_+(x)=a_0X^r+a_1X^{r+1}+a_2X^{r+2}+a_3X^{r+3}+a_4X^{r+4}+\dots$$

$a_0=1$

$$a_1=\frac{2a_0}{5}=\frac{2}{5}$$

$$a_2=\frac{2a_1}{5\cdot 2^2}=\frac{1}{10}\cdot\frac{2}{5}=\frac{2}{50}=\frac{1}{25},\quad \boxed{r=0}\rightarrow\text{(because of the question)}$$

$$y_+(x)=1+\frac{2}{5}x+\frac{1}{25}x^2.$$