HW 25 5.1

$$y'' - 3y' + 5y = -5\cos 3t, \quad y(0) = 3, \quad y'(0) = 1.$$

$$x' = Ax + b, \quad \text{where} \quad x = \begin{pmatrix} 4y \\ 2y' \end{pmatrix}$$

$$\begin{pmatrix} 4y' \\ 2y'' \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -\frac{10}{4} & 3 \end{pmatrix} \begin{pmatrix} 4y \\ 2y' \end{pmatrix} + \begin{pmatrix} 0 \\ -10\cos 3t \end{pmatrix}$$

$$y'' = 3y' - 5y - 5\cos 3t$$

$$2y'' = 6y' - 10y - 10\cos 3t$$

$$A = \begin{pmatrix} 0 & 2 \\ -\frac{10}{4} & 3 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ -10\cos 3t \end{pmatrix}$$

$$x(0) = \begin{pmatrix} 4y(0) \\ 2y'(0) \end{pmatrix} = \begin{pmatrix} 12 \\ 2 \end{pmatrix}$$

Hw 25. 5.1. pr. 7

$$X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, \quad A = \begin{pmatrix} 4 & -3 \\ 6 & -5 \end{pmatrix}, \quad X_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$X_1'' + a_1 X_1' + a_0 X_1 = 0$$

Remember: $a_1 = -\mathrm{Tr}(A)$, $a_0 = \det(A)$.

So if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\mathrm{tr}(A) = a+d$, $\det(A) = ad - bc$

So: $a_1 = -(4-5) = 1$, $a_0 = \det(A) = -20 + 18 = -2$.

$X_1(0) = 1$, and $\begin{pmatrix} X_1'(0) \\ X_2'(0) \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 6 & -5 \end{pmatrix} \overset{= \binom{x_1(0)}{x_2(0)}}{\begin{pmatrix} 1 \\ 1 \end{pmatrix}} = \begin{pmatrix} 4-3 \\ 6-5 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

So $X_1'(0) = 1$ and $X_2'(0) = 1$.

Similarly for X_2.