HW 29, 5.4.

$\lambda_\pm = \alpha \pm i\beta, \quad \alpha = 1, \quad \beta = 5.$

$v^\pm = a \pm b i, \quad a = -u^1, \quad b = u^2$

$$u^1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad u^2 = \begin{pmatrix} -1/2 \\ 1 \end{pmatrix}$$

$$x(t) = \frac{1}{2}\left(v^+ e^{\lambda_+ t} + v^- e^{\lambda_- t}\right)$$

Draw a phase portrait.

Solution

We need to find $x\left(-\frac{\pi}{2\beta}\right)$, and $x(0)$.

$$x(0) = \frac{1}{2}\left(v^+ + v^-\right) = a = -u_1$$

because $\frac{1}{2}\left(v^+ + v^-\right) = \frac{1}{2}\left(a + ib + a - ib\right) = \frac{1}{2}(a + a) = \overset{-u_1}{\overset{\parallel}{a}}$

$x\left(-\frac{\pi}{2\beta}\right) = ?$

[first fundamental solution from Nagy's lecture notes]

$$x(t) = x^1(t) = \left[a\cos(\beta t) - b\sin(\beta t)\right] e^{\alpha t} \quad \textcircled{=}$$

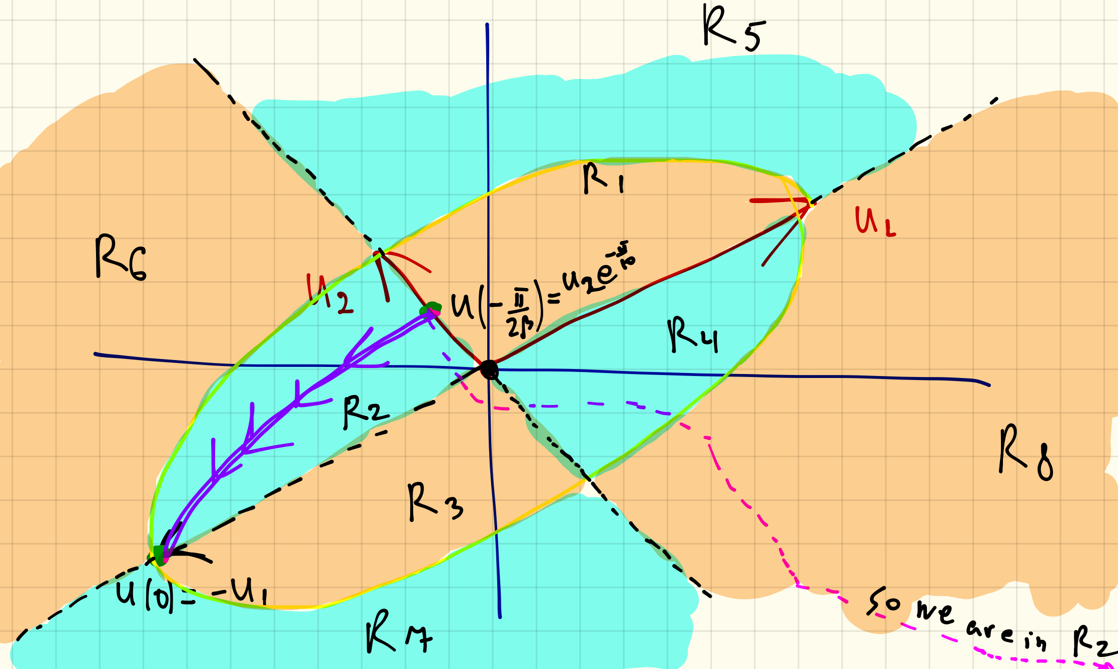
$$= \left[a\cos\left(\beta\cdot\left(-\frac{\pi}{2\beta}\right)\right) - b\sin\left(\beta\cdot\left(-\frac{\pi}{2\beta}\right)\right)\right] e^{\alpha t} =$$

$$= \left[a\cos\left(-\frac{\pi}{2}\right) - b\sin\left(-\frac{\pi}{2}\right)\right] e^{\alpha t} =$$

$$= b e^{\alpha t} = u_2 e^{-\frac{\pi}{10}}.$$

So $u(0) = -u_1$

$u\left(-\frac{\pi}{2\beta}\right) = u_2 e^{-\frac{\pi}{10}}$



therefore as time goes from $-\frac{\pi}{2\beta}$ to 0 we move like this

Hence after $t \in \left(0, \frac{\pi}{2\beta}\right)$ we will arrive in region R_7

We move counterclockwise.

When $t \to \infty$ we go far away from the origin. Hence $|x(t)| \to \infty$.

Since we always rotate, $\frac{x(t)}{|x(t)|} \to$ does not exist. Because this quantity measure rotation (it is an angle)

The answer is „None".

but $\lim_{t \to -\infty} |x(t)| = 0$ (because we approach to zero.

And $\lim_{t \to -\infty} \frac{x(t)}{|x(t)|} =$ „None".

And zeroth solution is unstable

Since we go far away from the origin.