$u = v(t)\cdot w(x)$ — always

$\partial_t u(t,x) = 3\boxed{\partial_x^2 u(t,x)}$ implies

$\dot v(t)\, w(x) = 3\, v(t)\, w''(x)$

$$\boxed{\frac{\dot v}{3v} = \frac{w''}{w} = -\lambda}$$

$e^{c_1 t + c_2} = e^{c_1 t}\cdot e^{c_2}$

① $\frac{\dot v}{3v} = -\lambda$ implies

$\dot v = -3\lambda v$ Hence $\frac{\dot v}{v} = -3\lambda$ $\quad \ln v = -3\lambda t + \tilde C$

$$\boxed{v = e^{-3\lambda t}\cdot C}$$

$C=1$ because $v(0)=1$

② $w'' = -\lambda w$ so $w'' + \lambda w = 0$ Hence

$r^2 + \lambda = 0$, $r = \pm i\sqrt{\lambda}$ (if $r = i, -i$

$w = \underline{c}\cdot\cos(\sqrt{\lambda}\cdot x) + \underline{d}\cdot\sin(\sqrt{\lambda}\cdot x)$

$w(0) = 0$ therefore $c = 0$

$\partial_x u(t,0) = 0$, $u = v(t)\cdot w(x)$, $v(t)\cdot w'(0) = 0$, $\boxed{w'(0)=0}$

$$w = \underline{\underline{c}} \cdot \cos(\sqrt{\lambda} \cdot x) + \underline{\underline{d}} \cdot \sin(\sqrt{\lambda} \cdot x)$$

$$w'(0) = 0 \qquad d = 0$$

$$w = \cos(\underline{\underline{\sqrt{\lambda}}} \cdot x)$$

$$u(t,3) = 0$$

$$v(t)\, w(3) \text{ implies } w(3) = 0$$

$$\cos(\sqrt{\lambda} \cdot 3) = 0, \quad \sqrt{\lambda} \cdot 3 = (2n-1)\frac{\pi}{2}$$

$$\sqrt{\lambda} = \frac{(2n-1)\pi}{6}, \quad \lambda = \left[\frac{(2n-1)\pi}{6}\right]^2$$

$$w_n = \cos\left[\left(\frac{2n-1}{6}\right)\pi x\right]$$

$$v_n = e^{-3\left[\frac{(2n-1)\pi}{6}\right]^2 t}$$

$$\boxed{c_n = \frac{\int_0^L f(x)\, w_n(x)\, dx}{\int_0^L w_n^2(x)\, dx}}$$

$$\operatorname{Re}\left[\alpha z_1 + (1-\alpha) z_2 - \beta z_3 - (1-\beta) z_4\right) = 0.$$

$$f(\omega) = \frac{A(\omega)}{A_0(\omega)} = \frac{A(\omega)\cdot\overline{A_0(\omega)}}{|A_0(\omega)|^2} =$$

$$\operatorname{Re}(A(\omega))\operatorname{Re}\left(\overline{A_0(\omega)}\right) - \operatorname{Im} A(\omega)\cdot \operatorname{I}\overline{A_0(\omega)} =$$

$$\operatorname{Re}(A(\omega))\operatorname{Re}(A_0(\omega)) + \operatorname{Im}(A(\omega))\cdot\operatorname{Im}(A_0(\omega))$$

$$a + wb = a_x + i a_y + (w_x + i w_y)(b_x + i b_y) =$$

$$a_x + w_x b_x - w_y b_y + i\left(a_y + w_y b_x + w_x b_y\right)$$

$$c_1 A\overline{A_0(\omega)} + c_2 A\overline{A_0(-\omega)} - d_1 A\overline{A_0(1)} - d_2 A\overline{A_0(-1)} = 0$$

$$d_1 \longmapsto (a_x + b_x)(a_x^0 + b_x^0) + (a_y + b_y)(a_y^0 + b_y^0)$$

$$d_2 \longmapsto (a_x - b_x)(a_x^0 - b_x^0) + (a_y - b_y)(a_y^0 - b_y^0)$$