Make sure you justify your statements completely and carefully.

1. Problem 3.1.8 from the **Notes**.

2. Problem 3.1.9 from the **Notes**.

3. Define the Hadamard product

$$\mathbf{x} * \mathbf{y} = (x_1 y_1, \dots, x_n y_n)$$

for the vectors $\mathbf{x} = (x_1, \dots, x_n)$ and $\mathbf{y} = (y_1, \dots, y_n)$ of F^n. For instance, over $\mathbb{F}_{13}$,

$$(1, 2, 3, 4) * (2, 3, 4, 5) = (2, 6, 12, 7)\,.$$

(a) Prove: the dot product $\mathbf{c} \cdot \mathbf{d}$ is the sum of the entries in $\mathbf{c} * \mathbf{d}$, and in particular for binary $\mathbf{c}$ and $\mathbf{d}$ (in $\mathbb{F}_2^n$) we have $\mathbf{c} \cdot \mathbf{d} = \mathrm{w_H}(\mathbf{c} * \mathbf{d}) \pmod 2$.

(b) Prove that, for vectors $\mathbf{x} = (x_1, \dots, x_n)$, $\mathbf{y} = (y_1, \dots, y_n)$ in the vector space F^n over the field F, we have

$$\mathrm{w_H}(x + y) \geq \mathrm{w_H}(x) + \mathrm{w_H}(y) - 2\,\mathrm{w_H}(x * y)\,.$$

Prove additionally that, for the binary field $F = \mathbb{F}_2$, we have equality:

$$\mathrm{w_H}(x + y) = \mathrm{w_H}(x) + \mathrm{w_H}(y) - 2\,\mathrm{w_H}(x * y)\,.$$

4. Problem 3.1.11 from the **Notes**. (Hint: The last part of the previous problem might be of help in part (a).)

5. (a) Give a syndrome dictionary for the $[8, 4]$ binary code C with the following check matrix:

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

(b) Use your dictionary to decode the received word:

$$(1, 0, 1, 0, 1, 1, 0, 0)\,.$$

(c) Use your dictionary to decode the received word:

$$(0, 1, 1, 1, 0, 0, 0, 0)\,.$$

(d) Use your dictionary to decode the received word:

$$(1, 1, 1, 0, 1, 1, 0, 1)\,.$$

6. Problem 4.1.5 from the **Notes**.

7. Problem 4.1.8 from the **Notes**.