Make sure you justify your statements completely and carefully.

1. Problem 5.1.3 from the **Notes**.

(HINT: The thing to realize initially is that, for a given  $\mathbf{c}$ , we can have both  $\mathbf{c} = \mathbf{ev}_{\alpha,v}(f(x)) \in GRS_{n,k}(\boldsymbol{\alpha}, \mathbf{v})$  and  $\mathbf{c} = \mathbf{ev}_{\beta,w}(g(x)) \in GRS_{n,k}(\boldsymbol{\beta}, \mathbf{w})$  for different polynomials f(x) and g(x).)

2. Problem A.2.26 from the Appendix 2 of the Notes. Prove first that

$$(a(x) + b(x))' = a'(x) + b'(x)$$

3. Problem 5.1.5 from the **Notes**. In parts (b) and (c) you may assume, for a finite field F with |F| = q, that the set of all elements of F is precisely the set of roots of the polynomial  $x^q - x$  and, hence, that the nonzero elements of F are precisely the roots of the polynomial  $x^{q-1} - 1$ . (We will prove this later in the course.)

4. Let C and D be linear codes over the field F with  $C = D^{\perp}$ . Let PC be the code C punctured at its last coordinate position, and let SD be the code D shortened at its last coordinate position. Prove that  $PC = SD^{\perp}$ .

REMARK. The code PC is constructed by deleting the last coordinate position from all codewords of C. The code SD is constructed by first selecting only those codewords of D that end in 0, and then deleting from these that final 0 position.

For the final two problems, 'justifying your statements completely and carefully' means you should show your calculations in enough detail that I can follow them (and can try to locate any mistakes). Please do not do your calculations on a machine. As these two problems are largely computational, I ask that you not collaborate on them.

5. Problem 5.2.5 from the **Notes**.

6. Consider the code  $GRS_{10,4}(\boldsymbol{\alpha}, \mathbf{v})$  over  $\mathbb{F}_{11}$  that was our classroom example. (a) Use Euclidean Algorithm decoding to decode the received vector

(b) Use Euclidean Algorithm decoding to decode the received vector

$$(1, 1, 1, 1, 0, 0, 0, 0, 0, 0)$$
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