Math 869: Final Assignment

Due Friday May 1 by 1:00pm

Instructions: I. Do not work with others on the exam.

II. You may use your notes and the book to prepare while studying for the exam but please do not use them while taking the exam.

III. Do not spend more than three hours working on the exam.

IV. Please either return it to my office (A-338 Wells Hall) or put in my mailbox which is on the third floor.

Problem 1. Construct a 2-dimensional CW complex with fundamental group $G \approx < a, b, c \mid a^2 b = 1, c^3 = 1 >$.

Problem 2. Find all the connected covering spaces of the topological space X that is the wedge of two real projective planes. That is $X = \mathbf{R}P^2 \vee \mathbf{R}P^2$. Which of these covering spaces are normal?

Problem 3. a) Prove that the spaces $X := S^1 \times S^1$ and $Y := S^1 \vee S^1 \vee S^2$ have isomorphic homology groups in all dimensions.

b) Prove that the universal coverings of X and Y do NOT have isomorphic homology groups in all dimensions.

Problem 4 Compute the homology groups of the 2-complex $X(m, n)$ obtained as quotient space of $S^1 \times S^1$ after the following identification: Identify points in the circle $S^1 \times \{x_0\}$ that differ by $\frac{2\pi}{m}$ rotation and identify points on the circle $\{x_0\} \times S^1$ that differ by $\frac{2\pi}{n}$ rotation.

Problem 5. For a topological space X and a subspace $A \subset X$ consider the short exact sequence of chain complexes

$$0 \to C_n(A) \xrightarrow{i} C_n(X) \xrightarrow{j} C_n(X, A) \to 0,$$

where the homomorphism i is induced by the inclusion $A \hookrightarrow X$ and j is the quotient map $C_n(X) \to C_n(X, A) := \frac{C_n(X)}{C_n(A)}$.

a) Explain why the short exact sequence above is split and thus we get $C_n(X) \approx C_n(A) \oplus C_n(X, A)$.

b) Does this splitting in part (a) always yield splittings $H_n(X) \approx H_n(A) \oplus H_n(X, A)$? If your answer is YES, prove your claim. If your answer is NO then explain why not and give an example to justify it.

Problem 6. Compute $H^i(\mathbf{R}P^2 \vee S^3; \frac{\mathbf{Z}}{2\mathbf{Z}})$, for all $i \geq 0$.

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Problem 7. a) Show that every finite cover of a closed , connected oriented manifold is oriented.

b) Is there a closed, connected, oriented 3-manifold M with $H_1(M) = \mathbf{Z} \oplus \mathbf{Z} \oplus \mathbf{Z}$ and $H_2(M) = \mathbf{Z}$? If your answer is NO then justify it using appropriate theorems. If your answer is YES then give an example of such a 3-manifold.