Math 869:Assignment 1

Due Friday February 6

Problem 1. Show that every homomorphism $\pi_1(S^1) \longrightarrow \pi_1(S^1)$ can be realized as the induced homomorphism ϕ_* of a map $\phi : S^1 \longrightarrow S^1$.

Problem 2. First observe that $\pi_1(X, x_0)$ can also be thought as the set of homotopy classes of maps $(S^1, s_0) \longrightarrow (X, x_0)$ that preserve the base-points. Let $\mathbf{M}(S^1, X)$ denote the set of homotopy classes of maps $S^1 \longrightarrow X$ with no conditions on base-points. Consider the natural map $\Phi : \pi_1(X, x_0) \longrightarrow \mathbf{M}(S^1, X)$ that is defined by "forgetting base points". Show that if X is path-connected then the following is true: $\Phi([f]) = \Phi([g]) \Longleftrightarrow [f]$ and $[g]$ are conjugate in $\pi_1(X, x_0)$.

Remark: The set $\mathbf{M}(S^1, X)$ is called the set of free homotopy-classes of loops in X. The problem above shows that for path-connected X the free homotopy classes of loops are in one-to-one correspondence with conjugacy classes in $\pi_1(X)$.

Problem 3. Consider the solid torus $X := S^1 \times D^2$ and $A := S^1 \times S^1$; the boundary torus. Show that there are no retractions $r : X \longrightarrow A$.

Problem 4. Let X be the quotient space of S^2 obtained by identifying the north and south poles into one point. Put a cell complex structure on it and use it to compute $\pi_1(X)$.

Problem 5. Solve Exercise 4 on page 53 on Hatcher's book.

Problem 6. Solve Exercise 12 on page 53 on Hatcher's book.