Math 869:Assignment 2

Due Friday March 6

Problem 1. Construct a simply connected covering space of the space $X \subset \mathbf{R^3}$ that is the union of a 2-sphere and a diameter of the 3-ball.

Problem 2. Show that if a path-connected, locally path-connected space X has finite $\pi_1(X)$, then every map $X \to S^1$ is nullhomotopic. [*Hint:* Use the covering space $\mathbf{R} \to S^1$.]

Problem 3. Let a, b be the generators of $\pi_1(S^1VS^1)$ corresponding to the two S^1 summands. Draw a picture of the covering space of S^1VS^1 corresponding to the normal subgroup generated by a^2, b^2 and $(ab)^4$, and PROVE that this covering space is indeed the correct one.

Problem 4. For a path-connected, locally path-connected and semi locally simply connected space X, call a path-connected covering space $\tilde{X} \to X$ ABELIAN if it is normal (regular) and has abelian deck transformation group. Show that X has an abelian covering space that is a covering space of any other abelian covering space of X and that such a "universal" abelian covering space is unique up to isomorphism. Describe this universal abelian cover explicitly for $X := S^1VS^1$.

Problem 5. Solve Exercise 20 on page 81 of Hatcher's book.

Problem 6. Solve Exercise 25 on page 81 of Hatcher's book.

Problem 7. Solve Exercise 7 on page 87 of Hatcher's book.