Homework Project 10/10 Solutions

5. Given a slope m, let $x(m)$ and $y(m)$ be the x- and y-intercepts of the line $y = m(x-3)+4$. To get $x(m)$, we solve $y = m(x-3)+4 = 0$, getting

$$x(m) = \frac{3m-4}{m}.$$

To get $y(m)$, we find the constant term in $y = m(x-3)+4 = mx - 3m + 4$, so

$$y(m) = -3m + 4.$$

Thus the length $L(m)$ of the diagonal line segment is:

$$\begin{aligned}
L(m) &= \sqrt{x(m)^2 + y(m)^2} \\
&= \sqrt{\left(\frac{3m-4}{m}\right)^2 + (-3m+4)^2} \\
&= \sqrt{\frac{(3m-4)^2}{m^2} + (3m-4)^2} && \text{since } (-a+b)^2 = (a-b)^2 \\
&= \sqrt{\frac{(3m-4)^2 + m^2(3m-4)^2}{m^2}} && \text{by combining fractions} \\
&= \sqrt{\frac{(3m-4)^2}{m^2}(m^2+1)} && \text{by factoring out } (3m-4)^2 \\
&= \left|\frac{3m-4}{m}\right| \sqrt{1+m^2} && \text{since } \sqrt{a^2} = |a| \\
&= \left(3 - \frac{4}{m}\right) \sqrt{1+m^2} && \text{since } m < 0, \text{ so } 3 - \tfrac{4}{m} > 0
\end{aligned}$$

7. Taking the derivative:

$$\begin{aligned}
L'(m) &= (3-4m^{-1})'\sqrt{1+m^2} + (3-4m^{-1})(\sqrt{1+m^2})' && \text{product rule} \\
&= 4m^{-2}\sqrt{m^2+1} + (3-4m^{-1})\frac{1}{2\sqrt{1+m^2}}(2m) && \text{chain rule} \\
&= \frac{4m^{-2}(m^2+1) + (3-4m^{-1})m}{\sqrt{1+m^2}} && \text{combine fractions} \\
&= \frac{4 + 4m^{-2} + 3m - 4}{\sqrt{1+m^2}} \\
&= \frac{4 + 3m^3}{m^2\sqrt{1+m^2}}
\end{aligned}$$

Now, to find the critical point where $L'(m) = 0$, we only need to set the numerator equal to 0:

$$4 + 3m^3 = 0 \quad \Longleftrightarrow \quad m = -\sqrt[3]{4/3} \cong -1.101$$

This is the desired minimum point. The minimum value is: $L\left(-\sqrt[3]{4/3}\right) \cong 9.866$.

8. To do the general problem, we just replace $(3, 4)$ with (a, b) throughout the above calculations. Thus we get:

$$L(m) = \left(a - \frac{b}{m}\right)\sqrt{1 + m^2}, \qquad L'(m) = \frac{b + am^3}{m^2\sqrt{1 + m^2}}\,.$$

The minumum point is thus $m = -\sqrt[3]{b/a}$, and the minimum value simplifies to the remarkable expression:

$$L\left(-\sqrt[3]{b/a}\right) = (b^{2/3} + a^{2/3})^{3/2}\,.$$

HW Proj 10/10 Solutions

Ques 6. L(m) = length of a line segment with slope m, passing through the point (3,4), cutting across the positive quadrant.

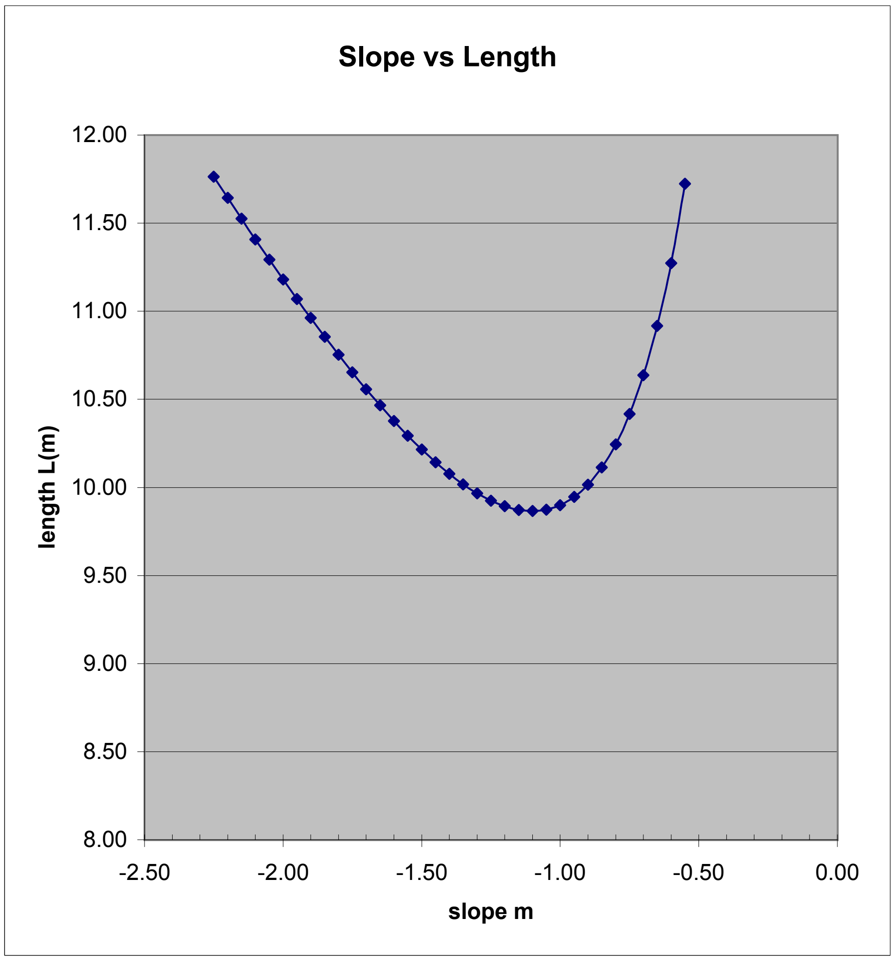
x(m) = x-intercept of line, y(m) = y-intercept of line

Formulas: $x(m) = 3-4/m$, $y(m) = -3m+4$

$$L(m) = \text{sqrt}(x(m)^2 + y(m)^2)$$

$$= \text{sqrt}((3 - 4/m)^2 + (-3m + 4)^2)$$

m	x(m)	y(m)	L(m)
-2.25	4.78	10.75	11.76
-2.20	4.82	10.60	11.64
-2.15	4.86	10.45	11.53
-2.10	4.90	10.30	11.41
-2.05	4.95	10.15	11.29
-2.00	5.00	10.00	11.18
-1.95	5.05	9.85	11.07
-1.90	5.11	9.70	10.96
-1.85	5.16	9.55	10.86
-1.80	5.22	9.40	10.75
-1.75	5.29	9.25	10.65
-1.70	5.35	9.10	10.56
-1.65	5.42	8.95	10.47
-1.60	5.50	8.80	10.38
-1.55	5.58	8.65	10.29
-1.50	5.67	8.50	10.22
-1.45	5.76	8.35	10.14
-1.40	5.86	8.20	10.08
-1.35	5.96	8.05	10.02
-1.30	6.08	7.90	9.97
-1.25	6.20	7.75	9.92
-1.20	6.33	7.60	9.89
-1.15	6.48	7.45	9.87
-1.10	6.64	7.30	9.87
-1.05	6.81	7.15	9.87
-1.00	7.00	7.00	9.90
-0.95	7.21	6.85	9.95
-0.90	7.44	6.70	10.02
-0.85	7.71	6.55	10.11
-0.80	8.00	6.40	10.24
-0.75	8.33	6.25	10.42
-0.70	8.71	6.10	10.64
-0.65	9.15	5.95	10.92
-0.60	9.67	5.80	11.27
-0.55	10.27	5.65	11.72



HW Proj 10/10 Solutions

Ques 6. L(m) = length of a line segment with slope m, passing through the point (3,4), cutting across the positive quadrant.

x(m) = x-intercept of line, y(m) = y-intercept of line

<u>Formulas</u>: x(m) = 3-4/m , y(m) = -3m+4

$$L(m) = \text{sqrt}(\ x(m)^2 + y(m)^2\ )$$

$$= \text{sqrt}(\ (\ 3 - 4/m\ )^2 + (\ -3m + 4\ )^2\ )$$

	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10	**m**	**x(m)**	**y(m)**	**L(m)**
11	-2.25	=3 -4/A11	=-3*A11 +4	=SQRT(B11^2+C11^2)
12	=A11+0.05	=3 -4/A12	=-3*A12 +4	=SQRT(B12^2+C12^2)
13	=A12+0.05	=3 -4/A13	=-3*A13 +4	=SQRT(B13^2+C13^2)
14	=A13+0.05	=3 -4/A14	=-3*A14 +4	=SQRT(B14^2+C14^2)
15	=A14+0.05	=3 -4/A15	=-3*A15 +4	=SQRT(B15^2+C15^2)
16	=A15+0.05	=3 -4/A16	=-3*A16 +4	=SQRT(B16^2+C16^2)
17	=A16+0.05	=3 -4/A17	=-3*A17 +4	=SQRT(B17^2+C17^2)
18	=A17+0.05	=3 -4/A18	=-3*A18 +4	=SQRT(B18^2+C18^2)
19	=A18+0.05	=3 -4/A19	=-3*A19 +4	=SQRT(B19^2+C19^2)
20	=A19+0.05	=3 -4/A20	=-3*A20 +4	=SQRT(B20^2+C20^2)
21	=A20+0.05	=3 -4/A21	=-3*A21 +4	=SQRT(B21^2+C21^2)
22	=A21+0.05	=3 -4/A22	=-3*A22 +4	=SQRT(B22^2+C22^2)
23	=A22+0.05	=3 -4/A23	=-3*A23 +4	=SQRT(B23^2+C23^2)
24	=A23+0.05	=3 -4/A24	=-3*A24 +4	=SQRT(B24^2+C24^2)
25	=A24+0.05	=3 -4/A25	=-3*A25 +4	=SQRT(B25^2+C25^2)
26	=A25+0.05	=3 -4/A26	=-3*A26 +4	=SQRT(B26^2+C26^2)
27	=A26+0.05	=3 -4/A27	=-3*A27 +4	=SQRT(B27^2+C27^2)
28	=A27+0.05	=3 -4/A28	=-3*A28 +4	=SQRT(B28^2+C28^2)
29	=A28+0.05	=3 -4/A29	=-3*A29 +4	=SQRT(B29^2+C29^2)
30	=A29+0.05	=3 -4/A30	=-3*A30 +4	=SQRT(B30^2+C30^2)
31	=A30+0.05	=3 -4/A31	=-3*A31 +4	=SQRT(B31^2+C31^2)
32	=A31+0.05	=3 -4/A32	=-3*A32 +4	=SQRT(B32^2+C32^2)
33	=A32+0.05	=3 -4/A33	=-3*A33 +4	=SQRT(B33^2+C33^2)
34	=A33+0.05	=3 -4/A34	=-3*A34 +4	=SQRT(B34^2+C34^2)
35	=A34+0.05	=3 -4/A35	=-3*A35 +4	=SQRT(B35^2+C35^2)
36	=A35+0.05	=3 -4/A36	=-3*A36 +4	=SQRT(B36^2+C36^2)
37	=A36+0.05	=3 -4/A37	=-3*A37 +4	=SQRT(B37^2+C37^2)
38	=A37+0.05	=3 -4/A38	=-3*A38 +4	=SQRT(B38^2+C38^2)
39	=A38+0.05	=3 -4/A39	=-3*A39 +4	=SQRT(B39^2+C39^2)
40	=A39+0.05	=3 -4/A40	=-3*A40 +4	=SQRT(B40^2+C40^2)
41	=A40+0.05	=3 -4/A41	=-3*A41 +4	=SQRT(B41^2+C41^2)
42	=A41+0.05	=3 -4/A42	=-3*A42 +4	=SQRT(B42^2+C42^2)
43	=A42+0.05	=3 -4/A43	=-3*A43 +4	=SQRT(B43^2+C43^2)
44	=A43+0.05	=3 -4/A44	=-3*A44 +4	=SQRT(B44^2+C44^2)
45	=A44+0.05	=3 -4/A45	=-3*A45 +4	=SQRT(B45^2+C45^2)