Math 132-12 & 16 **Fall 2003**

Homework Project
Due 10/10

Hand in answers to the questions below, including the required spreadsheets and graphs. See `http://www.math.msu.edu/~magyar/calculus` for corrections and updates. For technical help, see How to Graph with Excel, Introduction to Excel, and Homework Tips, all available on the course page.

Problem. (First physical version) We have a corner between two hallways, one 3ft wide, the other 4ft wide. We wish to move a long piece of furniture around the corner, without getting stuck. For simplicity, suppose the object is just a ladder with length L, but no width. What is the *longest* ladder which can be moved around the corner?

To answer this question, as for all word problems, we try to draw a picture and identify relevant variables geometrically. See the diagrams on the last page.

Problem. (Second physical version) We let the x and y axes represent the outer walls of the corner, and $(3,4)$ represent the inner corner point. The *longest* possible ladder will just fit through the *narrowest* part of the turn with its ends brushing against the x and y axes and its middle touching $(3,4)$. We must find the narrowest part of the turn.

Thus, consider all lines going through the point $(3,4)$. Which of these lines cuts a segment between the x and y axes whose length L is minimal?

To turn this into a math problem, we first identify *controlling variables* which define the line we are considering. The most natural is the *slope* m of the line: the point-slope formula tells us that a line going through $(3,4)$ with slope m has equation $y = m(x-3)+4$. (We could also define the line by specifying its x-intercept or its y-intercept, but we will think of these as variables depending on the slope.)

1. What is the independent variable which controls the orientation of the line? What is the dependent variable which we are trying to minimize?

2. Using a piece of graph paper and a ruler, make a rough estimate of the shortest line segment. (Note: The scale on the x and y axes must be the same as the scale for L.) Estimate the slope of the shortest line.

3. Now make a systematic measurement of the lengths: start with a line going from (0,10) through $(3,4)$ to the x-axis; then take lines more and more horizontal, down to the line from $(0,5)$. For each line segment, measure and record: (a) its length; (b) its x-intercept (run) and y-intercept (rise). Again, your length measurements must be in the same units for the line segment and for the axes.

Identify which of your lines is the shortest. How does it agree with your estimate?

4. Enter your measurements in a spreadsheet as four columns of data: for each line, the run, the rise, the slope $m =$ rise/run, and the length $L = L(m)$. That is, we consider L as a function of m. Note that since the lines are sloping down, the slopes m should be *negative.*

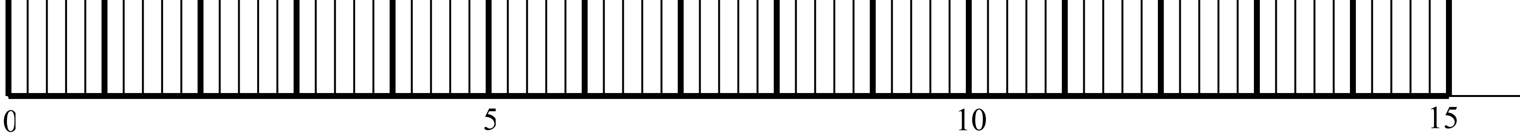
5. Find an algebraic formula for $L(m)$. Hint: Fix an unspecified slope m (so that m is constant for each line). Find the x-intercept $x(m)$ and the y-intercept $y(m)$ of the line, as functions of m. To get the x-intercept, you solve $m(x-3)+4=0$ for x in terms of m. To get the y-intercept, you find the constant term in the equation $y = m(x-3)+4 = 0$. Now $L(m)$ is the hypotenuse of the right triangle whose sides are $x(m)$ and $y(m)$.

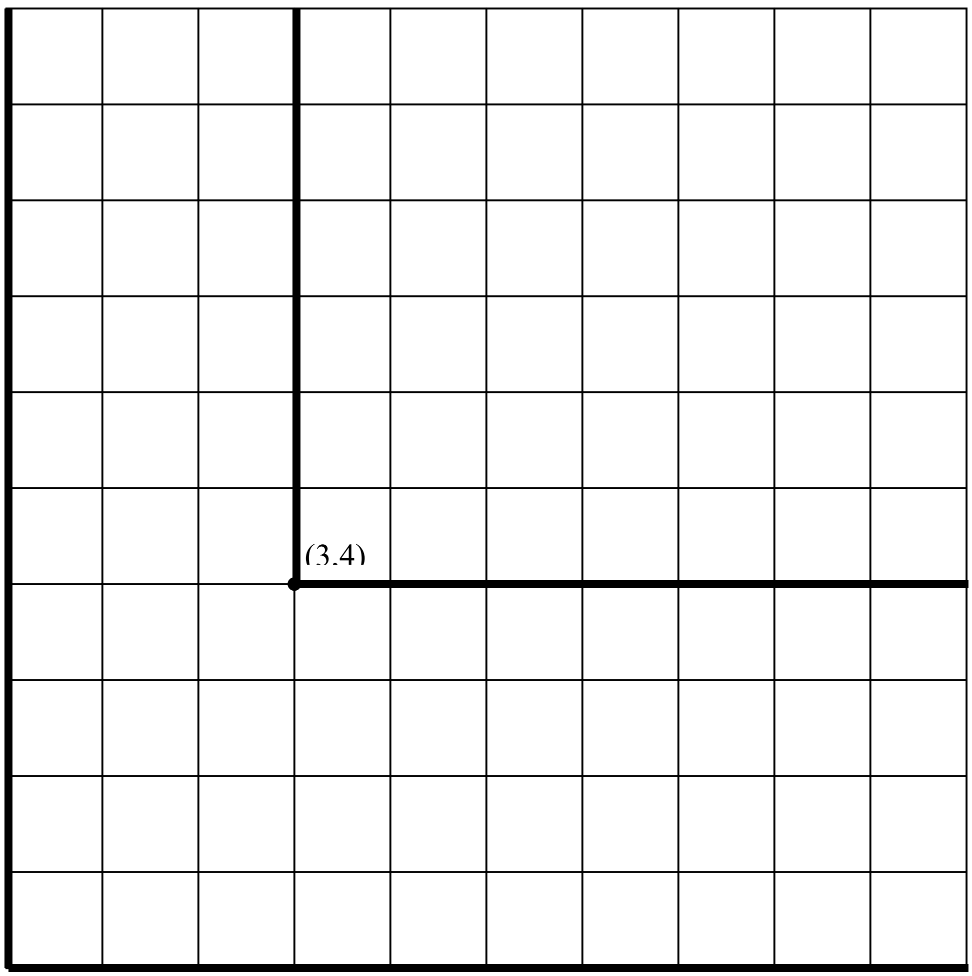
6. On the same spreadsheet as before, make a table and a graph of the algebraic formula for $L(m)$, approximately on the same interval of m-values as before. It should agree pretty closely with your physical measurements of $L(m)$.

From your graph and table, identify the slope m which yields the smallest length $L(m)$, up to 1 decimal place. Is this better than you could do by hand in Ques 3?

7. Find the derivative $L'(m)$ and solve $L'(m) = 0$. Compute the exact minimum value of $L(m)$. Evaluate this to 2 decimal places, and compare with your previous answers. Hint: The expression for $L'(m)$ is a much simpler if you first simplify $L(m)$ as much as possible. In particular, you can factor out $(3m-4)^2$ under the radical.

8. (Extra Credit) Now suppose that the hallways are of arbitrary width a and b, instead of 3 and 4. Repeat the algebraic part of the above assignment, computing a formula for $L(m)$ and solving $L'(m) = 0$. This will give a general formula for the minimum $L(m)$ in terms of a and b, and hence for the longest ladder fitting around *any* corner.





The walls of the hallway are marked by dark lines: the outer walls are the *x* and *y* axes, and the inner corner is at (3,4). A ladder can move around the corner if it fits through the <u>narrowest</u> part of the turn, so we are trying to find this narrowest part.

The ruler at the top is 15 units long, marked in increments of 0.2 units. You can cut off the ruler to do your measurements, or just use your own graph paper.

The narrowest part of the turn is somewhere between the extreme slopes.