The FALSE Distributive Law

The root of all evil — and the *source of 92% of all algebraic errors.*

The DISTRIBUTIVE LAW says:

$$\boxed{c \cdot (a+b) = c \cdot a + c \cdot b}$$

that is:

MULTIPLICATION distributes over addition.

IT DOES NOT SAY: "Everything in the whole universe distributes over addition." In fact, the horrible truth is:

Other than multiplication,
virtually **NOTHING** distributes over addition!

Do ROOTS distribute over addition? (Does $\sqrt{a+b} \stackrel{?}{=} \sqrt{a} + \sqrt{b}$?) NO!

Do POWERS distribute over addition? (Does $(a+b)^n \stackrel{?}{=} a^n + b^n$, $n \neq 1$?) NO!

Do RECIPROCALS distribute over addition? (Does $\frac{1}{a+b} \stackrel{?}{=} \frac{1}{a} + \frac{1}{b}$?) NO!

Do LOGS distribute over addition? (Does $\log(a+b) \stackrel{?}{=} \log a + \log b$?) NO!

Do TRIG FUNCTIONS distribute over addition? (Does $\sin(a+b) \stackrel{?}{=} \sin a + \sin b$?) NO!

Does ABSOLUTE VALUE distribute over addition? (Does $|a+b| \stackrel{?}{=} |a| + |b|$?) NO!

DO THESE THINGS AND DIE!

$\sqrt{x^2+25} \neq x+5$	$(x+y)^5 \neq x^5+y^5$	$\frac{1}{x+3} \neq \frac{1}{x} + \frac{1}{3}$	$\log(3+x) \neq \log 3 + \log x$
$\sqrt[4]{a^8-1} \neq a^2-1$	$(d-2)^3 \neq d^3-8$	$\frac{x}{x-y} \neq 1 - \frac{x}{y}$	$\sin(x+\pi) \neq \sin x + \sin \pi$
$\sqrt[3]{s^3+t^3} \neq s+t$	$(\sqrt{x}+7)^2 \neq x+49$	$\frac{1}{\frac{1}{x}+5} \neq x + \frac{1}{5}$	$\lvert x+10\rvert \neq \lvert x\rvert + 10$

A Related Matter: Solving Equations.

What you do to one *side* of an equation, you must to the the other *side.*

But: *side* means the *whole side:* **do not distribute** what you're doing to each individual piece of the side.

Example: Given the equation $\sqrt{x} + 6 = x$. Suppose you square both sides; you get: $x + 36 \stackrel{?}{=} x^2$. **WRONG!** You are saying $(\sqrt{x}+6)^2 \stackrel{?}{=} x + 36$. **WRONG!**
You get: $(\sqrt{x}+6)^2 \stackrel{!}{=} x^2 \implies x + 12\sqrt{x} + 36 = x^2$. **RIGHT!**

Example: Given $\frac{1}{x} = 5 + \frac{1}{\sqrt{y}}$. You get: $x \stackrel{?}{=} \frac{1}{5} + \sqrt{y}$. **WRONG!** You get: $x \stackrel{!}{=} \frac{1}{5+\frac{1}{\sqrt{y}}}$. **RIGHT!**