Two-Dimensional Calculus

Summary of the kinds of functions we have covered, how to visualize them, their derivatives and integrals.

function	picture	approximation	derivative	integral theorem
$f:\mathbb{R}\to\mathbb{R}$ $f(x)$	graph $y=f(x)$ derivative vect field	$f(x)\approx f(x_\circ)+f'(x_\circ)(x-x_\circ)$	tangent slope $[Df_{x_\circ}]=[f'(x_\circ)]$	*Second Fund Thm of Calc* $\int_a^b f'(x)\,dx=f(b)-f(a)$
$f:\mathbb{R}^2\to\mathbb{R}$ $f(x,y)$	graph $z=f(x,y)$ contour map $f(x,y)=c$ gradient vector field ∇f	$f(x,y)\approx f(x_\circ,y_\circ)+\nabla f(x_\circ,y_\circ)\cdot(x-x_\circ,y-y_\circ)$	gradient vector $[\,\nabla f(x_\circ,y_\circ)\,]=\begin{bmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\end{bmatrix}$	*Gradient Theorem* $\int \nabla f(\vec{c})\cdot d\vec{c}=f(\vec{c}(1))-f(\vec{c}(0))$
$\vec{c}:\mathbb{R}\to\mathbb{R}^2$ $\vec{c}(t)=(x(t),y(t))$ $a\le t\le b$	parametrized curve	$\vec{c}(t)\approx\vec{c}(t_\circ)+\vec{c}\,'(t_\circ)(t-t_\circ)$	tangent vector $[\,\vec{c}\,'(t_0)\,]=\begin{bmatrix}x'(t_\circ)\\ y'(t_\circ)\end{bmatrix}$	*Length Formula* $\int_a^b \lvert\vec{c}\,'(t)\rvert\,dt=\mathrm{len}(\vec{c})$
$G:\mathbb{R}^2\to\mathbb{R}^2$ $G(u,v)=(x(u,v),y(u,v))$	uv-grid in xy-plane parametrized region $D=G(D^*)$	$G(u,v)\approx G(u_\circ,v_\circ)+DG_{(u_\circ,v_\circ)}(u-u_\circ,v-v_\circ)$	Jacobian matrix $[\,DG_{(u_\circ,v_\circ)}\,]=\begin{bmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{bmatrix}$	*Substitution Formula* $\iint_D f(x,y)\,dx\,dy=$ $\iint_{D^*} f(G(u,v))\,\lvert\det DG\rvert\,du\,dv$
$\vec{F}:\mathbb{R}^2\to\mathbb{R}^2$ $\vec{F}(x,y)=(p(x,y),q(x,y))$	field of arrows in xy-plane		rate of circulation $\mathrm{curl}\,\vec{F}=\frac{\partial q}{\partial x}-\frac{\partial p}{\partial y}$ rate of flux $\mathrm{div}\,\vec{F}=\frac{\partial p}{\partial x}+\frac{\partial q}{\partial y}$	*Curl Theorem* $\iint_D \mathrm{curl}\,\vec{F}(x,y)\,dx\,dy$ $=\oint \vec{F}(\vec{c})\cdot d\vec{c}$ *Divergence Theorem* $\iint_D \mathrm{div}\,\vec{F}(x,y)\,dx\,dy$ $=\oint \vec{F}(\vec{c})\cdot d\vec{n}$ $\vec{c}=$ ctr-clock boundary of D $d\vec{c}=(x'(t),y'(t))\,dt$ $\vec{n}=$ outward normal of D $d\vec{n}=(y'(t),-x'(t))\,dt$