Homework: Group Axioms

Define a group by the set:

$$G = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is a bijective function}\},$$

and the operation $*$ as the composition of functions. That is, the group product of two functions is their composite function:

$$f * g = f \circ g = h, \quad \text{where } h(x) = f(g(x)).$$

Show that this G with this operation satisfies the four group axioms. You may use facts about about functions that we have previously covered.

AXIOM 1. *Closure:* $\forall f, g \in G,\ f \circ g \in G.$
Explain briefly; no need for a detailed proof.

AXIOM 2. *Associativity:* $\forall f, g, h \in G,\ (f \circ g) \circ h = f \circ (g \circ h)$
Show this by checking: $((f \circ g) \circ h)(x) = (f \circ (g \circ h))(x)$ for all $x \in \mathbb{R}$.

AXIOM 3. *Identity element:* $\exists\, e{\in}G, \forall f{\in}G,\ f \circ e = e \circ f = f.$
What function $e : \mathbb{R} \to \mathbb{R}$ acts as the identity element? Why? Is it an element of G?

AXIOM 4. *Inverse element:* $\forall f{\in}G, \exists\, g{\in}G, f \circ g = g \circ f = e.$
What function corresponds to the inverse element of f? Is it an element of G? Why?

Does the above group satisfy the following additional property? Prove your answer.
Commutativity: $\forall f, g \in G, f \circ g = g \circ f$?