PROBLEM 1. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a fence. With 800m of fence available, what shape of rectangle will enlcose the largest area?

Solution A.

$$\text{Area} = ab$$

$$2a + b = 800$$

$$b = 800 - 2a$$

$$f(a) = a(800 - 2a)$$

$$f'(a) = 800 - 4a = 0 \implies a = 200, b = 400$$

Solution B. Flesh out the above to make it readable, in the style of Houston Chapter 3.

PROBLEM 2. Prove the following:

> THEOREM: *Among all rectangles with a given fixed area, the one with the smallest perimeter is a square.*

Proof A.

$$xy = A$$

$$y = \frac{A}{x}$$

$$f(x) = 2x + \frac{2A}{x}$$

$$f'(x) = 2 - \tfrac{2A}{x^2} = 0 \implies x = \sqrt{A} \implies y = \sqrt{A} \implies \text{square}$$

Proof B. Again, flesh out the above into a full proof.