Math 299 **Recitation 3** **Sept 12, 2013**

1. Let the real functions f and g be defined by:

$$f(x) = \sqrt{x} \quad \text{and} \quad g(x) = 1 - x^2.$$

a. Find the largest domain for f so that $f \circ g$ is a real function? Write your answer using proper notation and briefly explain why the domain can't be larger.

b. Find a possible codomain for the function f.

2. Consider the function $t : \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z} \times \mathbb{Z}$ defined by $t(a, b) = (5a + 2b, 3a + b)$. Write a formula for the inverse function $s : \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z} \times \mathbb{Z}$, and verify it is indeed an inverse. Can you conclude that t is injective or surjective?

3. Let $j \leq m$ be natural numbers. Recall that $[m] = \{1, 2, \ldots, m\}$, and $[m] \backslash \{j\}$ means $[m]$ with the element j removed. Consider the bijection $h : [m] \backslash \{j\} \longrightarrow [m-1]$ given by:

$$h(k) = \begin{cases} k & \text{for } k < j \\ k - 1 & \text{for } k > j. \end{cases}$$

Problem: Give a formula for the inverse function $g : [m-1] \rightarrow [m] \backslash \{j\}$.

4. The natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$ seem to be a smaller set than the integers $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, though it is not clear what "smaller" means for infinte sets. In fact it is possible to find a bijective function $f : \mathbb{N} \rightarrow \mathbb{Z}$.

Problem: Find such a function f, and write it explicitly, either in cases or as a formula using the floor function $\lfloor \ \rfloor$.

5. Consider $A = \{1, 2, 3\}$ and $B = \{w, x, y, z\}$. Justify your answers by citing a previous result or by finding a pattern in the list of all possibilities.

a. How many injective functions are there from A to B?

b. How many surjective functions are there from B to A?