Math 299 **Recitation 4** **Sept. 19, 2013**

PROBLEM 1. Use truth tables to prove that the statements "not (A and B)" and "(not A) or (not B)" are equivalent.

A	B	not(A)	not(B)	A and B	not(A and B)	not(A) or not(B)
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

PROBLEM 2. (**Bonus problem** - can be left for homework.) Use Problem 1 to prove that "not (A or B)" and "(not (A) and not (B))" are equivalent **without writing out truth tables**.

Note that if A is equvalent to B then not(A) is equivalent to not(B) and vice versa. In Problem 1 we showed that "not (A and B)" and "(not A) or (not B)" are equivalent. Therefore, their negations are equivalent. Namely, "A and B" is equivalent to "not((not A) or (not B))".

Therefore, "C and D" is equvalent to "not(not(C) or not(D))". Here we only changed the names of the variables.

Now apply the above to the statement C substituted with not(A) and the statement D substituted with not(B).

PROBLEM 3. Use *not*, *and* and *or* to define the following set operations.

(a) X^c (b) $A\bigcap B$ (c) $A\bigcup B$ (d) $A\setminus B$ (e) $(A\bigcap B)^c$ (f) $A^c\bigcup B^c$ (g) $(A\bigcup B)^c$

SOLUTION:

(a) The statement "$x\in X^c$" is equivalent to the statement "*not*$(x\in X)$."

(b) The statement "$x\in A\bigcap B$" is equivalent to the statement "$x\in A$ and $x\in B$."

(c) The statement "$x\in A\bigcup B$" is equivalent to the statement "$x\in A$ or $x\in B$."

(d) The statement "$x\in A\setminus B$" is equivalent to the statement "$x\in A$ and *not*$(x\in B)$."

(e) The statement "$x\in (A\bigcap B)^c$" is equivalent to the statement "*not*$(x\in A$ and $x\in B)$."

(f) The statement "$x\in A^c\bigcup B^c$" is equivalent to the statement "*not*$(x\in A)$ or *not*$(x\in B)$."

(g) The statement "$x\in (A\bigcup B)^c$" is equivalent to the statement "not$(x\in A$ or $x\in B)$."

PROBLEM 4. (**Bonus problem** - can be left for homework.) Combine Problems 1-3 to prove De Morgan's laws for sets.

(a) $(A\bigcap B)^c = A^c\bigcup B^c$

SKETCH OF SOLUTION: Using the work from Problem 3e, one can see that $x\in (A\bigcap B)^c$ is equivalent to "not$(x\in A$ and $x\in B)$". Now, according to Problem 1, this is equivalent to "not$(x\in A)$ or not$(x\in B)$", which, according to Problem 3f, is equivalent to $x\in A^c\bigcup B^c$.

(b) $(A \bigcup B)^c = A^c \bigcap B^c$ (Uses Problem 2.)

PROBLEM 5. Clearly state the assumption and conclusion in each statement. Negate the following.

(a) If $f'(x)$ is positive on (a, b), then $f(b) > f(a)$.

(b) The quadratic equation $ax^2 + bx + c = 0$ (with $a, b, c \in \mathbb{R}$) has two real solutions, assuming its discriminant is positive.

(c) The set $\{x \in \mathbb{R} \,|\, x^2 + a = 0\}$ is nonempty only if $a \leq 0$.

(d) For a function f to be continuous at a point c, it is necessary that $\lim_{x \to c-} f(x) = \lim_{x \to c+} f(x)$.

(e) For $x = c$ to be a vertical asymptote, it is sufficient that $\lim_{x \to c+} f(x) = -\infty$.

PROBLEM 6. Use truth tables to show that "$A \Rightarrow B$" is equivalent to "$not(B) \Rightarrow not(A)$" and is not equivalent to "$not(A) \Rightarrow not(B)$." Note that the second statement is called the **contrapositive** of the first. We will revisit this later.

A	B	not(A)	not(B)	A $\Rightarrow$ B	not(B) $\Rightarrow$ not(A)	not(A) $\Rightarrow$ not(B)
T	T					
T	F					
F	T					
F	F					