Recovery of Compactly Supported Functions from Spectrogram Measurements via Lifting

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2017
Friday, July 7^{th}, 2017

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Motivation

Applications: The phase-retrieval problem arises whenever the detectors can only capture *intensity* measurements. For example,

X-ray crystallography
Diffraction imaging
Ptychographic Imaging
...

Our goals: Approaching realistic measurement designs compatible with, e.g., ptychography, coupled with computationally efficient and robust recovery algorithms.

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Motivating Application

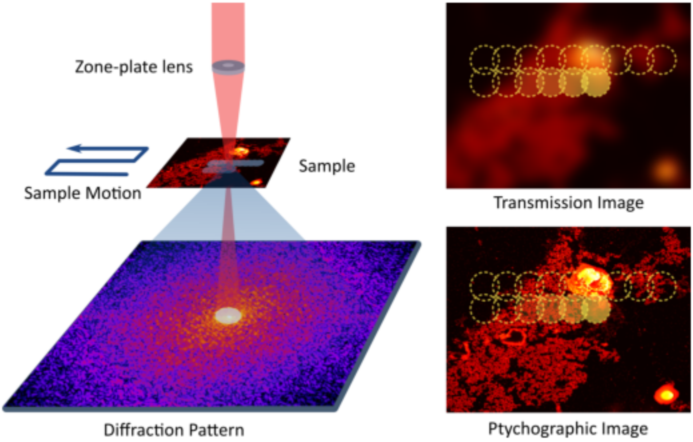

Figure: Ptychographic Imaging

Algorithms for Discrete Phase Retrieval

- There has been a good deal of work on signal recovery from phaseless STFT measurements in the *discrete setting*:
  - First f and g are modeled as vectors ab initio,
  - Then recovered from discrete STFT magnitude measurements.

- Recovery techniques include
  - Iterative methods (Alt. Proj. for STFT) along the lines of Griffin and Lim [8, 12],
  - Alternating Projections [7],
  - Graph theoretic methods for Gabor frames based on polarization [11, 9],
  - Semidefinite relaxation-based methods [5], and others [2, 1, 4, 3].

Signal Recovery from STFT Measurements

- In 1-D ptychography [10, 7], a compactly supported specimen $f:\mathbb{R}\to\mathbb{C}$, is scanned by a focused beam $g:\mathbb{R}\to\mathbb{C}$ which translates across the specimen in fixed overlapping shifts $l_1,\ldots,l_K\in\mathbb{R}$.
- At each such shift a phaseless diffraction image is sampled by a detector.
- The measurements are modeled as STFT magnitude measurements:

$$b_{k,j} := \left|\int_{-\infty}^{\infty} f(t)\, g(t-l_k)\, e^{-2\pi i\omega_j t} dt\right|^2,\ 1\le k\le K,\ 1\le j\le N. \quad (1)$$

- We aim to approximate f (up to a global phase) using these $b_{k,j}$ measurements.

- Given stacked spectrogram samples from (1),

$$\vec{b} = \left(\; b_{1,1}, \ldots, b_{1,N}, b_{2,1}, \ldots, b_{K,N} \; \right)^T \in [0,\infty)^{NK}, \tag{2}$$

  approximately recover a piecewise smooth and compactly supported function $f : \mathbb{R} \to \mathbb{C}$ up to a global phase.

- WLOG assume that the support of f is contained in $[-1,1]$.

- Motivated by ptychography, we primarily consider the beam function g to also be (effectively) compactly supported within $[-a,a] \subsetneq [-1,1]$.

- Assume also that g is smooth enough that its Fourier transform decays relatively rapidly in frequency space compared to $\hat{f}$. Examples of such g include both Gaussians, as well as compactly supported C^∞ bump functions [6].

- Using techniques from [4, 3] on discrete PR adapted to continuous PR, recover samples of $\hat{f}$ at frequencies in $\Omega = \{\omega_1, \ldots, \omega_N\}$, giving $\vec{f} \in \mathbb{C}^N$ with $f_j = \widehat{f}(\omega_j)$:
  - First, a truncated lifted linear system is inverted in order to learn a portion of the rank-one matrix $\vec{f}\vec{f}^*$.
  - Then, angular synchronization is used to recover $\vec{f}$ from the portion of $\vec{f}\vec{f}^*$ above.
- Reconstruct $\widehat{f}$ via standard sampling theorems.
- Invert this approximation in order to learn f.
- This linear system is banded and Toeplitz, with band size determined by the decay of $\hat{g}$: if g is effectively bandlimited to $[-\delta, \delta]$ the computational cost is $\mathcal{O}\left(\delta N(\log N + \delta^2)\right)$ - **essentially FFT-time** in N for small δ.

Lifted Formulation

Lemma (Lifting Lemma)

Suppose $f : \mathbb{R} \to \mathbb{C}$ *is piecewise smooth and compactly supported in* $[-1,1]$. *Let* $g \in L^2([-a,a])$ *be supported in* $[-a,a] \subset [-1,1]$ *for some* $a < 1$, *with* $\|g\|_{L^2} = 1$. *Then for all* $\omega \in \mathbb{R}$,

$$|\mathcal{F}[f \cdot S_l g](\omega)| = \frac{1}{2}\left|\sum_{m \in \mathbb{Z}} e^{-\pi i l m} \hat{f}\left(\frac{m}{2}\right) \hat{g}\left(\frac{m}{2} - \omega\right)\right|$$

for all shifts $l \in [a-1, 1-a]$.

Proof.

- Plancherel's Theorem implies that

$$\left|\mathcal{F}\left[f\cdot S_l g\right](\omega)\right|^2=\left|\int_{-\infty}^{\infty}\hat{f}\left(\omega-\eta\right)\hat{g}\left(-\eta\right)e^{-2\pi i l\eta}d\eta\right|^2.$$

- Applying Shannon's Sampling theorem to $\hat{f}$ and recalling that $\mathcal{F}\left[f\star g\right]=\hat{f}\hat{g}$ yield

$$\begin{aligned}\left|\mathcal{F}\left[f\cdot S_l g\right](\omega)\right|^2&=\left|\sum_{m\in\mathbb{Z}}\hat{f}\left(\frac{m}{2}\right)\left[\hat{g}\left(\cdot\right)e^{-2\pi i l(\cdot)}\star\mathrm{sinc}\pi\left(m+2\left(\cdot\right)\right)\right]\left(-\omega\right)\right|^2\\&=\frac{1}{4}\left|\sum_{m\in\mathbb{Z}}\hat{f}\left(\frac{m}{2}\right)e^{-\pi i l(m-2\omega)}\int_{-l-1}^{-l+1}g\left(u\right)e^{-2\pi i u\left(\frac{m}{2}-\omega\right)}du\right|^2.\end{aligned}$$

- The result follows by noting the support of g and the Fourier type integral in the last equality.

□

Lifted Form

We write our measurements in a lifted form

$$\left|\mathcal{F}\left[f \cdot S_l g\right](\omega)\right|^2 \approx \frac{1}{4} \vec{X}_l^* \vec{Y}_\omega \vec{Y}_\omega^* \vec{X}_l$$

where $\vec{X}_l \in \mathbb{C}^{4\delta+1}$ and $\vec{Y}_\omega \in \mathbb{C}^{4\delta+1}$ are the vectors

$$\vec{X}_l = \begin{pmatrix} e^{\pi i l(2\delta)} \hat{g}\left(-\delta\right) \\ e^{\pi i l(2\delta-1)} \hat{g}\left(\frac{1}{2}-\delta\right) \\ \vdots \\ e^{\pi i l \cdot 0} \hat{g}\left(0\right) \\ \vdots \\ e^{\pi i l(1-2\delta)} \hat{g}\left(\delta-\frac{1}{2}\right) \\ e^{\pi i l(-2\delta)} \hat{g}\left(\delta\right) \end{pmatrix}, \vec{Y}_\omega = \begin{pmatrix} \overline{\hat{f}\left(\omega-\delta\right)} \\ \overline{\hat{f}\left(\omega-\delta+\frac{1}{2}\right)} \\ \vdots \\ \overline{\hat{f}\left(\omega\right)} \\ \vdots \\ \overline{\hat{f}\left(\omega+\delta-\frac{1}{2}\right)} \\ \overline{\hat{f}\left(\omega+\delta\right)} \end{pmatrix}.$$

Lifted Form

We write our measurements in a lifted form

$$|\mathcal{F}[f \cdot S_l g](\omega)|^2 \approx \frac{1}{4} \vec{X}_l^* \vec{Y}_\omega \vec{Y}_\omega^* \vec{X}_l$$

Here, $\vec{Y}_\omega \vec{Y}_\omega^*$ is the rank-one matrix

$$\begin{bmatrix} \left|\hat{f}(\omega-\delta)\right|^2 & \cdots & \overline{\hat{f}(\omega-\delta)}\hat{f}(\omega) & \cdots & \overline{\hat{f}(\omega-\delta)}\hat{f}(\omega+\delta) \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \overline{\hat{f}(\omega)}\hat{f}(\omega-\delta) & \cdots & \left|\hat{f}(\omega)\right|^2 & \cdots & \overline{\hat{f}(\omega)}\hat{f}(\omega+\delta) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{\hat{f}(\omega+\delta)}\hat{f}(\omega-\delta) & \cdots & \overline{\hat{f}(\omega+\delta)}\hat{f}(\omega) & \cdots & \left|\hat{f}(\omega+\delta)\right|^2 \end{bmatrix}.$$

Note the occurrence of the magnitudes of $\hat{f}$ on the diagonal, and the relative phase terms elsewhere.

- Let $\mathbf{F} \in \mathbb{C}^{N \times N}$ be defined as

$$\mathbf{F}_{i,j} = \begin{cases} \overline{\hat{f}\left(\frac{i-2n-1}{2}\right)} \hat{f}\left(\frac{j-2n-1}{2}\right), & \text{if } |i-j| \leq 2\delta, \\ 0, & \text{otherwise,} \end{cases}, \text{ where } n = \frac{N-1}{4}.$$

- $\mathbf{F}$ is composed of overlapping segments of matrices $\vec{Y}_\omega \vec{Y}_\omega^*$ for $\omega \in \{-n, \ldots, n\}$.

- Thus, our spectrogram measurements can be written as

$$\vec{b} \approx \text{diag}(\mathbf{G}\mathbf{F}\mathbf{G}^*), \tag{3}$$

where $\mathbf{G} \in \mathbb{C}^{NK \times N}$ is a block Toeplitz matrix encoding the $\vec{X}_l$'s.

- We consistently vectorize (3) to obtain a linear system which can be inverted to learn $\vec{F}$, a vectorized version of $\mathbf{F}$.

- In particular, we have
$$\vec{b} \approx \mathbf{M}\vec{F}, \tag{4}$$
where $\mathbf{M} \in \mathbb{C}^{NK \times N^2}$ is computed by passing the canonical basis of $\mathbb{C}^{N \times N}$ through (3).
- We solve the linear system (4) as a least squares problem.
- Experiments have shown that M is of rank NK.
- The process of recovering the Fourier samples of f from $\vec{F}$ is known as angular synchronization.

Angular Synchronization

- Angular synchronization is the process recovering d angles $\phi_1, \phi_2, \ldots, \phi_d \in [0, 2\pi)$ given noisy and possibly incomplete difference measurements of the form

$$\phi_{ij} := \phi_i - \phi_j, \quad (i,j) \in \{1,2,\ldots,d\} \times \{1,2,\ldots,d\}.$$

- We are interested in angular synchronization problems that arise when performing ***phase retrieval*** from ***local correlation measurements*** [4, 3].

Leading Eigenvector $\leftrightarrow$ Phase Vector for $\delta = 1/2$

$$\left[|\vec{f}_1|^2 \;\; \vec{f}_1\vec{f}_2^* \;\; \vec{f}_2\vec{f}_1^* \;\; |\vec{f}_2|^2 \;\; \vec{f}_2\vec{f}_3^* \;\; \vec{f}_3\vec{f}_2^* \;\; |\vec{f}_3|^2 \;\; \vec{f}_3\vec{f}_4^* \;\; \vec{f}_4\vec{f}_3^* \;\; |\vec{f}_4|^2 \right]^T$$

$$\Big\downarrow \text{(re-arrange)}$$

$$\begin{bmatrix} |\vec{f}_1|^2 & \vec{f}_1\vec{f}_2^* & 0 & 0 \\ \vec{f}_2\vec{f}_1^* & |\vec{f}_2|^2 & \vec{f}_2\vec{f}_3^* & 0 \\ 0 & \vec{f}_3\vec{f}_2^* & |\vec{f}_3|^2 & \vec{f}_3\vec{f}_4^* \\ 0 & 0 & \vec{f}_4\vec{f}_3^* & |\vec{f}_4|^2 \end{bmatrix} \quad (\mathbf{F}, 4\delta+1 \text{ entries in band})$$

$$\Big\downarrow \text{(normalize entrywise)}$$

$$\begin{bmatrix} 1 & e^{\mathbb{i}(\phi_1-\phi_2)} & 0 & 0 \\ e^{\mathbb{i}(\phi_2-\phi_1)} & 1 & e^{\mathbb{i}(\phi_2-\phi_3)} & 0 \\ 0 & e^{\mathbb{i}(\phi_3-\phi_2)} & 1 & e^{\mathbb{i}(\phi_3-\phi_4)} \\ 0 & 0 & e^{\mathbb{i}(\phi_4-\phi_3)} & 1 \end{bmatrix}$$

$$\Big\downarrow \text{(top eigenvector)}$$

$$\approx \left[\mathbb{e}^{\mathbb{i}\phi_1} \;\; \mathbb{e}^{\mathbb{i}\phi_2} \;\; \mathbb{e}^{\mathbb{i}\phi_3} \;\; \mathbb{e}^{\mathbb{i}\phi_4}\right]^T$$

(Reconstruction of $\hat{f}$ samples) $\quad \left[|\vec{f}_1|e^{\mathbb{i}\,\phi_1} \quad |\vec{f}_2|e^{\mathbb{i}\,\phi_2} \quad |\vec{f}_3|e^{\mathbb{i}\,\phi_3} \quad |\vec{f}_4|e^{\mathbb{i}\,\phi_4}\right]^T$

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($\mathbf{F}, 4\delta + 1$ entries in band)

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$$\begin{bmatrix} 1 & e^{\mathbb{i}(\phi_1-\phi_2)} & 0 & 0 \\ e^{\mathbb{i}(\phi_2-\phi_1)} & 1 & e^{\mathbb{i}(\phi_2-\phi_3)} & 0 \\ 0 & e^{\mathbb{i}(\phi_3-\phi_2)} & 1 & e^{\mathbb{i}(\phi_3-\phi_4)} \\ 0 & 0 & e^{\mathbb{i}(\phi_4-\phi_3)} & 1 \end{bmatrix}$$

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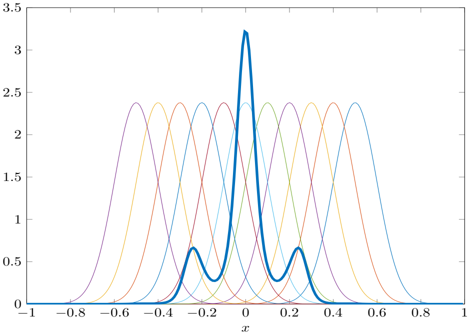
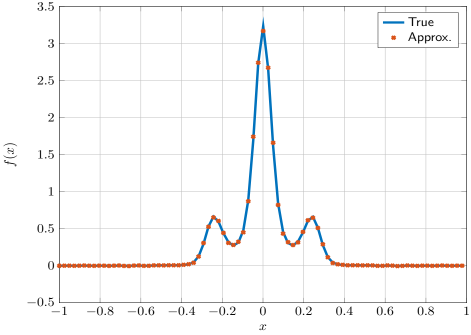
- Consider the Oscillatory Gaussian $f(x) = 2^{\frac{1}{4}} e^{-8\pi x^2} \cos(24x) \chi_{[-1,1]}$.
- Take as window the Gaussian $g(x) = c \cdot e^{-16\pi x^2} \chi_{\left[-\frac{1}{2},\frac{1}{2}\right]}$.



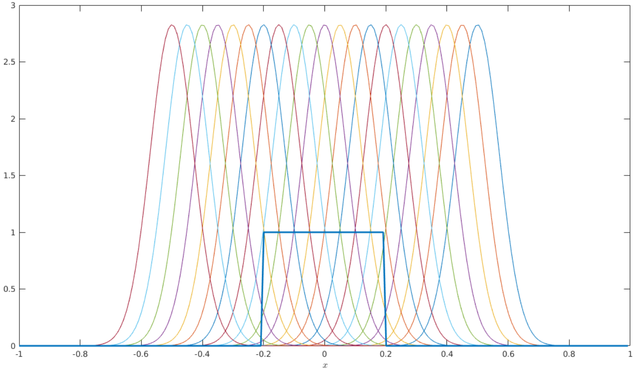
- We use a total of 11 shifts of g, and choose 61 values of ω from $[-15, 15]$ sampled in half-steps, and set $\delta = 7$.

The reconstruction in physical space is shown at selected grid points in the figure below.

The relative ℓ^2 error in physical space is 1.872×10^{-2}.



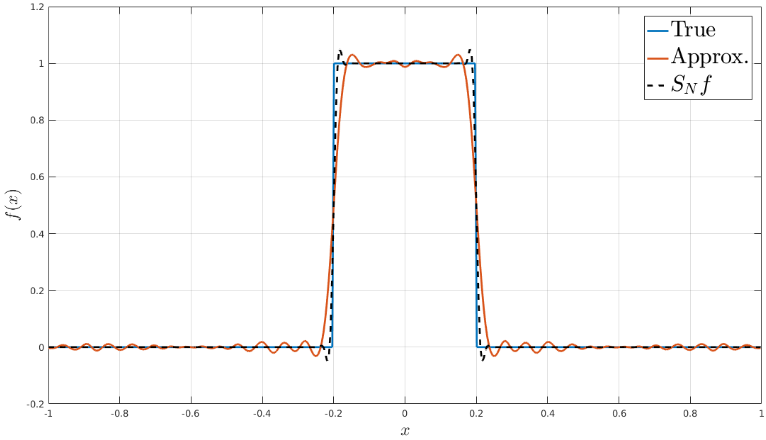
- Consider the Characteristic Function $f(x) = \chi_{[-\frac{1}{5},\frac{1}{5}]}$.
- Take as window the Gaussian $g(x) = c \cdot e^{-32\pi x^2} \chi_{[-\frac{1}{2},\frac{1}{2}]}$.



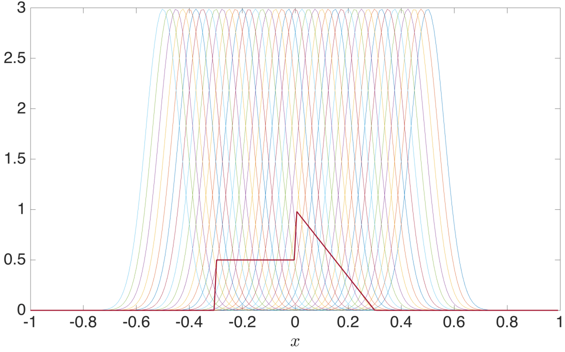
- We use a total of 21 shifts of g, and choose 293 values of ω from $[-73, 73]$ sampled in half-steps, and set $\delta = 10$.

The reconstruction in physical space is shown in the figure below.

The relative ℓ^2 error in physical space is 1.509×10^{-1}.



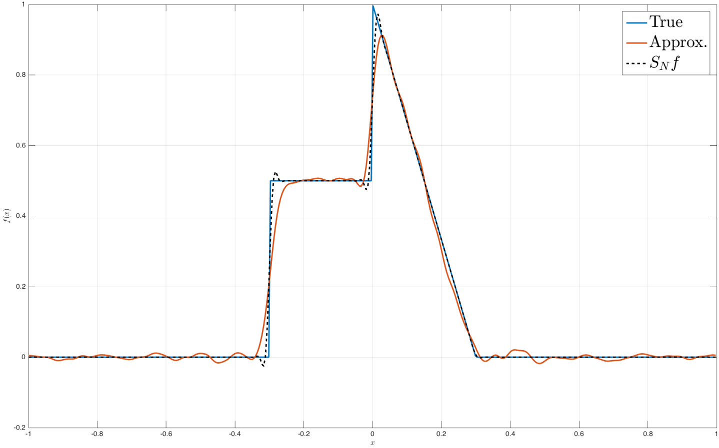
- Consider the Peicewise Smooth Function $f(x) = \frac{1}{2}\chi_{[-\frac{3}{20},0]} + \left(\frac{-10}{3}x+1\right)\chi_{[0,\frac{3}{20}]}$.
- Take as window the Gaussian $g(x) = c \cdot e^{-32\pi x^2/5}\chi_{[-\frac{1}{2},\frac{1}{2}]}$.



- We use a total of 41 shifts of g, and choose 281 values of ω from $[-70, 70]$ sampled in half-steps, and set $\delta = 10$.

The reconstruction in physical space is shown in the figure below.

The relative ℓ^2 error in physical space is 1.162×10^{-1}.



References I

[1] T. Bendory and Y. C. Eldar. Non-convex phase retrieval from STFT measurements. 2016. preprint, arXiv:1607.08218.

[2] Y. C. Eldar, P. Sidorenko, D. G. Mixon, S. Barel, and O. Cohen. Sparse phase retrieval from short-time Fourier measurements. *IEEE Signal Process. Lett.*, 22(5):638–642, 2015.

[3] M. A. Iwen, B. Preskitt, R. Saab, and A. Viswanathan. Phase retrieval from local measurements: Improved robustness via eigenvector-based angular synchronization. 2016. preprint, arXiv:1612.01182.

[4] M. A. Iwen, A. Viswanathan, and Y. Wang. Fast phase retrieval from local correlation measurements. *SIAM J. Imaging Sci.*, 9(4):1655–1688, 2016.

[5] K. Jaganathan, Y. C. Eldar, and B. Hassibi. STFT phase retrieval: Uniqueness guarantees and recovery algorithms. *IEEE J. Sel. Topics Signal Process.*, 10(4):770–781, 2016.

References II

[6] S. G. Johnson. Saddle-point integration of C^∞ “bump" functions. 2015. preprint, arXiv:1508.04376.

[7] S. Marchesini, Y.-C. Tu, and H.-t. Wu. Alternating projection, ptychographic imaging and phase synchronization. *Appl. Comput. Harmon. Anal.*, 41(3):815–851, 2016.

[8] S. Nawab, T. Quatieri, and J. Lim. Signal reconstruction from short-time Fourier transform magnitude. *IEEE Trans. Acoust., Speech, Signal Process.*, 31(4):986–998, 1983.

[9] G. E. Pfander and P. Salanevich. Robust phase retrieval algorithm for time-frequency structured measurements. 2016. preprint, arXiv:1611.02540.

[10] J. Rodenburg, A. Hurst, and A. Cullis. Transmission microscopy without lenses for objects of unlimited size. *Ultramicroscopy*, 107(2):227–231, 2007.

References III

[11] P. Salanevich and G. E. Pfander. Polarization based phase retrieval for time-frequency structured measurements. In *Proc. 2015 Int. Conf. Sampling Theory and Applications (SampTA)*, pages 187–191, 2015.

[12] N. Sturmel and L. Daudet. Signal reconstruction from STFT magnitude: A state of the art. In *Int. Conf. Digital Audio Effects (DAFx)*, pages 375–386, 2011.