

American Options

(i) The American Call has the same value as the American Put, thus we need not price the American Call.

(ii) Price American Put in 2 step example

~~Payoff at any time~~ Payoff at any time

$H(t), H^{\omega_1}, H^{\omega_1, \omega_2}$ given.

$$\text{ie } H(t) = H^{\omega_1, \dots, \omega_t} \begin{cases} H(t) = H^{\omega_1} \\ H(t) = H^{\omega_1, \omega_2} \end{cases}$$

$$H(t) = (X - S(t))^+.$$

At time step 2, $H(t) = (X - S(t))^+$ only since we cannot hold onto it any longer ($H(3) = 0$).

∴ Value @ time 2 is $V(t) = H(t)$

Time step 1 value of holding onto claim,

$$\frac{1}{1+r} E^*(V(t) | \mathcal{F}_1) = \frac{1}{1+r} E^*(V(t) | S(1))$$

∴ if $H(t) \geq \frac{1}{1+r} E^*(V(t) | \mathcal{F}_1)$ We should exercise

$H(t) < \frac{1}{1+r} E^*(V(t) | \mathcal{F}_1)$ Hold onto Put

$$\therefore V(t) = \max \left\{ H(t), \frac{1}{1+r} E^*(V(t) | \mathcal{F}_1) \right\}$$

Iterate to find $V(t)$.

Note payoff of American put $(X - S(t))^+$

decreases as S increases

∴ if S values only increase we should exercise immediately.

If S values ^{max} decrease on the other hand, the put insures against possible loss of value.

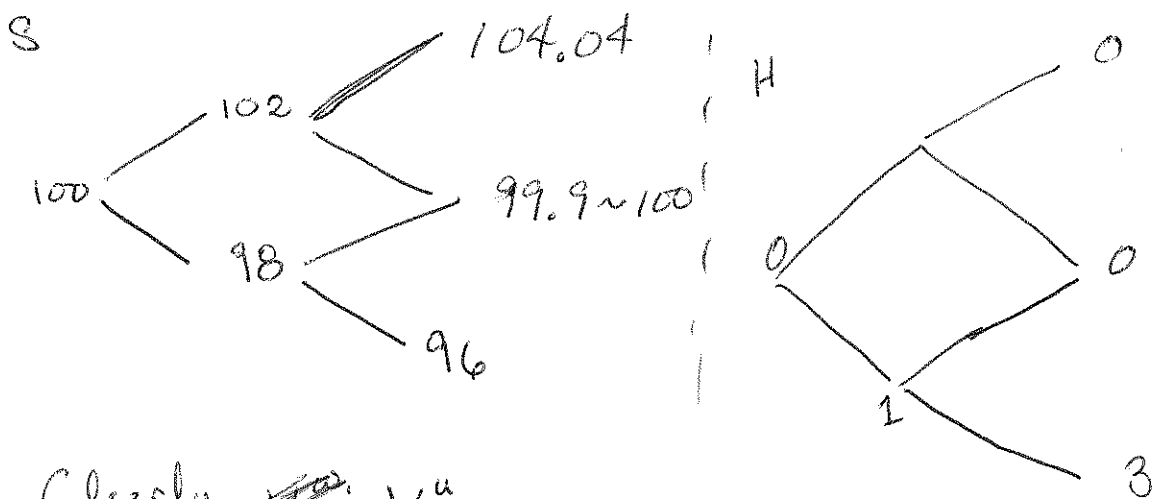
∴ we must take $m_1 > 0 > m_2$.

American Put.

$$S_0 = 100, \quad m_1 = 0.02, \quad m_2 = -0.02$$

$$r = 0.01$$

$$X = 99$$



Clearly ~~not~~ $V^u = 0$

But for B_d do we hold or exercise early?

$$V_{(1)}^d = \max \left\{ E^* \left\{ \frac{S_{(2)}}{1+r} \mid B_d \right\}, 1 \right\}$$

$$P_d^* = \frac{1}{4}, \quad P_u^* = \frac{3}{4}$$

$$= \left(\frac{1}{1.01} \cdot \frac{1}{4} \cdot 3 \right) V_{(1)} = 1 \quad \therefore \text{exercise early.}$$

Value @ zero

$$E^* \frac{V_{(1)}}{1.01} = \frac{1}{4.04}$$

Same example but set $r=0$

$$m_1 = .02, m_2 = -.02$$

$$\text{again } V^u = 0, P_u^* = \frac{1}{2}, P_d^* = \frac{1}{2}.$$

$$V^d = \left(\frac{1}{2} \cdot 3\right) \vee 1 = 3/2.$$

\therefore hold option

Value @ time zero

$$V(0) = m_2 \times \left\{ 0, \frac{1}{1+r} E^* V(1) \right\}$$

$$= \frac{1}{2} \cdot \frac{3}{2} = 3/4.$$

\therefore Exercise early in cases where interest on payoff is enough to be more valuable than value from holding onto option.

Also value of put decreases as interest increases.