American Options

* American CALL

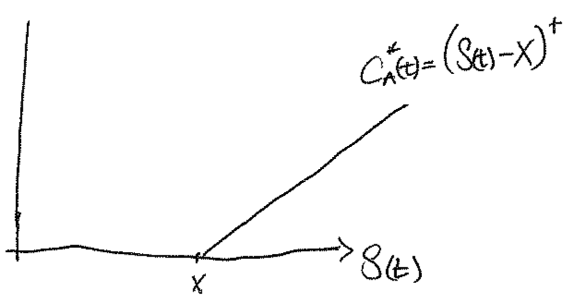
Contract which allows holder to purchase security at anytime t, $0 < t < T$ for the strike price X.

* American Put

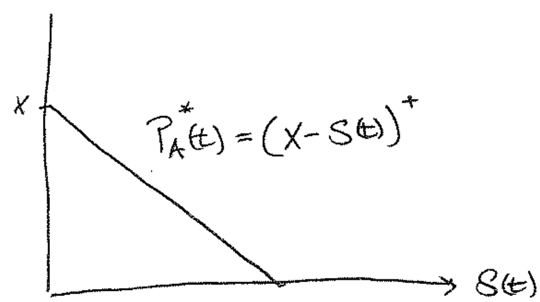
Contract which allows holder to sell security at anytime t, $0 < t < T$ for the strike price X.

Intrinsic Value:

Call:



Put



Clearly: $C_A(t) \ge C_E(t)$; & $P_A(t) \ge P_E(t)$.

Since American Options allow more freedom than European Options.

PC Parity:

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Follow strategies from Euro Call.

(i) $t = 0$ * Short Call

* Long Put + SECURITY.

We cannot control Exercise time of Call

- exercise put @same time.

Portfolio $(\$, C_A, P_A) = (C_A - P_A - S(0), -1, 1)$.

(ii) τ, $0 < \tau \leq T$, exercise

~~* Sell share for X~~

~~* Clear Call + Put = Collect~~

* Sell share for X $(X > S(\tau))$

Collect: $V_\tau = X + e^{r\tau}(C_A - P_A - S(0))$.

By No Arbitrage $V_\tau \leq 0$

$\therefore \quad C_A - P_A \leq S(0) - e^{-\tau r} X$

We Only know this is true for some $0 < \tau < T$

$\therefore \quad C_A - P_A \leq S(0) - e^{-Tr} X$.

II PC parity:

(i) $t=0$ * Short Put + Security

* Long Call.

We cannot control time of Exercise for Put.

- Exercise Call @ same time

Portfolio ~ $(\$, C_A, P_A) = (P_A + S(0) - C_A, 1, -1)$

(ii) τ, $0 < \tau \leq T$ exercise

~~* Purchase Security for X → Clears Call~~

~~* Sell Security for X → C~~

* Purchase Security for X → Clears Put

* ~~Sell~~ Return Security to Owner
→ Clears Short Security.

Collect:

$$V_\tau = e^{r\tau}(S(0) + P_A - C_A) - X$$

No arbitrage: $V_\tau \leq 0$

∴ Since we only know this for <u>some</u> τ.

$$P_A(0) - C_A(0) \leq X - S(0).$$

The value of the American call is the same as the value of the European CALL.

$$C_A(t) = C_E(t)$$

This is Equivalent to the statement,

One should <u>never</u> exercise the American Call early.

In fact, the statement extends to any option which has Intrinsic value 0 if $S(t) = 0$ & is convex & increasing in $S(t)$.

$\therefore$ If you are holding an American Option & want to get rid of it you would sell it on the market rather than exercise it.

To show $C_A = C_E$ we only need to show $C_A \leq C_E$.

No-Arbitrage proof:

$\{t=0\}$ * Short American Call

* Long European Call.

portfolio: $(\$, C_A, C_E) = (-C_E + C_A, -1, 1)$

$t=\tau$ $\{0<\tau<T\}$ Exercise time of American Call.

* Short Share (borrow security)

+ Collect X

$t=T$ * Exercise Euro call - buy share for X

- Return share to owner.

$$V_T = (C_A - C_E)\frac{1}{B(0,T)} + X\frac{1}{B(\tau,T)} - X \leq 0$$

$$\therefore \quad C_A(0) \leq C_E(0) + X\left(1 - \frac{1}{B(\tau,T)}\right)B(0,T)$$

$\underline{\underline{\tau = T}} \Rightarrow C_A(0) \leq C_E(0).$ ✓

$\tau < T \Rightarrow C_A(0) < C_E(0)$ ↯