Pricing American Put

* American Call = ~~American~~ Euro Call
  - this is because American Call should not be excersized early
    But sometimes American Put should be Excersized early –

Intrinsic value of American Put:

$$H_t = (X - S_t)^+$$

Given expiry T we have

$$V_T = H_T = (X - S_T)^+ .$$

Solve backward to find previous values.

Discrete Model: (Binomial)

Let N be the expiry step —

~~[illegible]~~ At time $N-1$ we know results of First $N-1$ steps

~~[illegible]~~ $\omega \in A \Rightarrow \omega_i =$

$$\underline{\omega}_k = \{ \nu \in \Omega : \nu_i = \omega_i \text{ for } i = 1, \dots, k \}$$

(i) If we excercize @ time $N-1$ we obtain:

$$H(N-1, \underline{\omega}_{N-1})$$

(ii) If we hold onto the option our expected payoff is

$$G(\underline{\omega}_{N-1}) = \frac{1}{1+r} \tilde{E}(V_N \mid \underline{\omega}_{N-1})$$

~~[illegible]~~ Exercise if intrinsic value H exceeds the expectation of future value

$\therefore$

$$V_{N-1}^{\underline{\omega}_{N-1}} = \max\left\{ H_{(N-1, \underline{\omega}_{N-1})} , \frac{1}{1+r} \tilde{E}\left(V_N \mid \underline{\omega}_{N-1}\right) \right\}$$

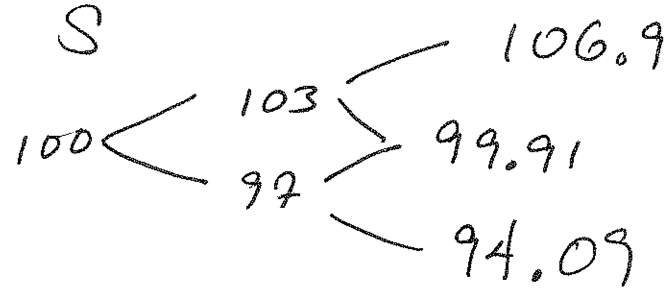
<u>or</u>

$$V_{N-1} = \max\left\{ H(\mathcal{F}_{N-1}) , \frac{1}{1+r} \tilde{E}(V_N \mid \mathcal{F}_{N-1}) \right\}$$

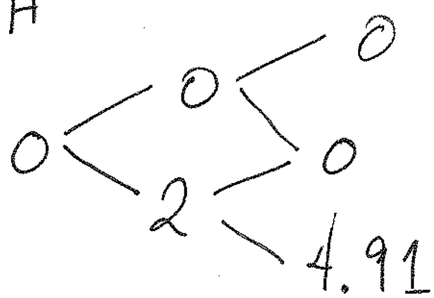
$S_0 = 100$; $r = 0$; $m_u = .03$; $m_d = -.03$

$\tilde{p}_u = \tilde{p}_d = 1/2$; $X = 99$; $H = (X - S)^+$

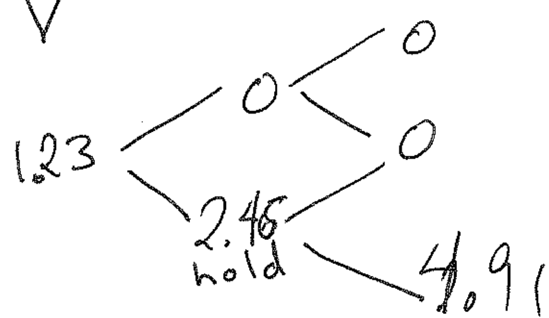
S



H



V



$$V^u = \max\{H^u, \mathbb{E}\{V_2 \mid u\}\} = 0$$

$$V^d = \max\{H^d = 2, \mathbb{E}\{V_2 \mid d\} = \tfrac{1}{2} 4.91\} = 2.455$$

$$V_0 = \max\{H_0 = 0, \mathbb{E}(V_1) = (1/2)(2.455)\} = 1.2275$$

$$V_0 = \$1.23$$

(4)

$S_0 = 100$; $r = .01$; $m_u = .03$; $m_d = -.03$

$\tilde{p}_u = \frac{2}{3}$, $\tilde{p}_d = \frac{1}{3}$; $X = 99$; $H = (X - S)^+$

Same S + H values

H

$$\begin{tikzcd}[column sep=small, row sep=tiny]
 & & 0 \\
 & 0 \arrow[ur] \arrow[dr] & \\
0 \arrow[ur] \arrow[dr] & & 0 \\
 & 2 \arrow[ur] \arrow[dr] & \\
 & & 4.91
\end{tikzcd}$$

S

$$\begin{tikzcd}[column sep=small, row sep=tiny]
 & & 106.9 \\
 & 103 \arrow[ur] \arrow[dr] & \\
100 \arrow[ur] \arrow[dr] & & 99.91 \\
 & 97 \arrow[ur] \arrow[dr] & \\
 & & 94.09
\end{tikzcd}$$

V

$$\begin{tikzcd}[column sep=small, row sep=tiny]
 & & 0 \\
 & 0 \arrow[ur] \arrow[dr] & \\
.66 \arrow[ur] \arrow[dr] & & 0 \\
 & \underset{\text{exercise}}{2} \arrow[ur] \arrow[dr] & \\
 & & 4.91
\end{tikzcd}$$

$V^u = 0$

$V^d = \max\left(H^d = 2,\ \frac{1}{1.01} E\{V_1 | d\}\right) =$

$= \max(2, 1.62) = 2$

$\therefore$ increased interest $\Rightarrow$ early exercise.

+ decreases value of put.