Exam 1 Review

1. Consider a market with 3 securities. Suppose $S_1(0) = 150,\ S_2(0) = 100\ S_3(0) = 200.$

$$S_1(1) = 140 + X;\ S_2(1) = 90 + Y;\ S_3(1) = 190 + Z$$

Suppose the density of X, Y, Z is given by

$$f_{X,Y,Z}(x, y, z) = c(x + 2y + 3z) \text{ if } x, y, x \geq 0 \text{ and } x + 2y + 3z \leq 30$$

   a. Find the value of the normalizing constant c.
   b. Find the joint distribution of return values of the securities K_1, K_2, K_3.
   c. Find the distribution of K_1 given K_2.
   d. Find the risk and return. Find the Covariance matrix
   e. Find the minimal risk porfolio.
   f. What is the maximal bond rate so that a market portfolio exists?
   g. Does the min variance portofolio require short selling?

2.) Suppose a bond pays coupons of value C every quarter, (at times $t = 1/4, 2/4, 3/4, ..$), the face value of the bond $F = \$500$ is paid at maturity of 2 years.

   – If the effective interest rate is 2% what value should the coupons be so that the bond is at par?
   – What is the value of the bond at $t = 1/12$? $t = 4/12$? $t = 5/12$?

3.) Let $V(t)$ be the value of a coupon bond at time t which pays \$ 100 monthly at an effective interest of 7%

   a) Suppose the bond matures in 1 year at which time it pays a face value (along with the final coupon) of \$ 10,000, is it above below or at par? What is $V(0)$? What is $V(1/4 + 0)$?
   b) Is it preferable to purchase the bond priced at 7% continuous interest?