

Homework 1

1. Consider an experiment involving flipping 4 coins in a row.
 - a. What is the sample space of the experiment? Using the notation from the notes, recall $\mathcal{F}_1 = \{\emptyset, A_T, A_H, \Omega\}$ write down all elements of \mathcal{F}_2 .
2. Consider an experiment involving flipping an infinite sequence of coins.
 - a. Using notation from the notes, show $\mathcal{F} = \cup_{i=1}^{\infty} \mathcal{F}_i$ is a σ - algebra.

To show this, note that any $A, B \in \mathcal{F}$ belong to \mathcal{F}_i for some i . Verify the conditions of the definition of σ -algebra to complete the problem.

3. For $0 < a < b$, consider X given by density.

$$f_X(x) = \begin{cases} cx & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

- a. For any given a, b find c .
 - b. Find the Expectation and Variance for this random variable.
4. Consider $Z = (X, Y)$ given by density.

$$f_{X,Y}(x, y) = \begin{cases} c(xy) & x \geq 0; y \geq 0; x + 2y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- a. Find c .
- b. Find the marginals for random variables X, Y .
- c. Find the expectation and variances of X, Y .
- d. Find the covariance of X and Y .
- e. Find the density $f_Y(y|X = 1)$.
- f. Find $E(X|Y)$ and $E(Y|X)$.
- g. Let $W = X + 2Y$ find $f_X(x|W)$.