

Homework 6 - Quiz

Name: KEY

1. Consider a 1 step binomial model of security with $S(0) = 30$ and $r = .1$ and $\mathbb{P}(S(1) = 35) = 1/2$ and $\mathbb{P}(S(1) = 30) = 1/2$.

i - Find the Risk neutral measure. 8

ii - What is the $\tau \equiv$ Radon-Nikodym Derivative of the original probability to the risk neutral measure? 2

iii - Value a European Call with Expiry $t = 1$ and strike price $X = 34$. 6

iv - What are the holdings of the replicating portfolio of the call? 2

v - Use the Put Call parity equation to find the value of the European put with Expiry 1 and strike price $X = 34$. 2

$$(i) \quad \tilde{p}_u = \frac{33-30}{35-30} = \frac{3}{5} \quad ; \quad \tilde{p}_d = \frac{35-33}{35-30} = \frac{2}{5}$$

$$(ii) \quad \frac{\tilde{p}_u}{p_u} = \tau(u) = \frac{3/5}{1/2} = \frac{6}{5} \quad ; \quad \frac{\tilde{p}_d}{p_d} = \tau(d) = \frac{2/5}{1/2} = \frac{4}{5}$$

$$(iii) \quad C_E(u) = (S_1^u - X)^+ = (35 - 34)^+ = 1.$$

$$C_E(d) = (S_1^d - X)^+ = (30 - 34)^+ = 0$$

$$C_E(0) = \frac{1}{1.1} \mathbb{E}\{C_E^{(1)}\} = \frac{1}{1.1} \left\{ 1 \cdot \frac{3}{5} + 0 \cdot \frac{2}{5} \right\} = .5454...$$

$$(iv) \quad x_0 = \frac{C^u - C^d}{S^u - S^d} = \frac{1}{5} \quad ; \quad y_0 = \frac{C^d S^u - C^u S^d}{(1.1)(S^u - S^d)}$$

$$y_0 = \frac{0 \cdot 35 - 1 \cdot 30}{(1.1)(35 - 30)} = -5.45...$$

$$(v) \quad C - P = F = S_0(1.1) - X = -1.$$

$$P_0 = C + 1 = 1.5454...$$