HW4

1.) $g(x) = x^T b = x_1 b_1 + \cdots + x_n b_n$

$\frac{\partial}{\partial x_i} g = b_i$

(a) $\hookrightarrow \nabla g = b = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$

$f(x) = x^T A x = \sum_{ij} x_i A_{ij} x_j$

$\frac{\partial}{\partial x_i} f = 2 x_i A_{ii} + \sum_{j \neq i} A_{ij} x_j + \sum_{j \neq i} A_{ji} x_i$

$\hookrightarrow \frac{\partial}{\partial x_i} f = (Ax)_i + (A^T x)_i$

(b) $\therefore \nabla f = Ax + A^T x$

2.)

Ω	K_1	K_2	K_3	$\mathbb{P}$
ω_1	.2	0	-.1	1/4
ω_2	.1	.1	0	1/4
ω_3	0	.1	.2	1/4
ω_4	-.1	.1	0	1/4

3)

$$\mu = \begin{pmatrix} \mathbb{E}K_1 \\ \mathbb{E}K_2 \\ \mathbb{E}K_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \frac{1}{40}$$

$$\Sigma = \begin{pmatrix} 20 & -6 & -10 \\ -6 & 3 & 5 \\ -10 & 5 & 19 \end{pmatrix} \frac{1}{1600}$$

4) $\Sigma_{12}^{-1} = \begin{pmatrix} 3 & 6 \\ 6 & 20 \end{pmatrix} \frac{1600}{24}$ $\Sigma_{13}^{-1} = \begin{pmatrix} 19 & 10 \\ 10 & 20 \end{pmatrix} \frac{1600}{356}$

$\Sigma_{23}^{-1} = \begin{pmatrix} 19 & -5 \\ -5 & 3 \end{pmatrix} \frac{1600}{32}$

Min Variance Portfolio:

$w_{12} = \begin{pmatrix} 9 \\ 26 \end{pmatrix} \frac{1}{35}$; $w_{13} = \begin{pmatrix} 29 \\ 30 \end{pmatrix} \frac{1}{59}$; $w_{23} = \begin{pmatrix} 14 \\ -2 \end{pmatrix} \frac{1}{12}$

Min Risk

Risk & return:

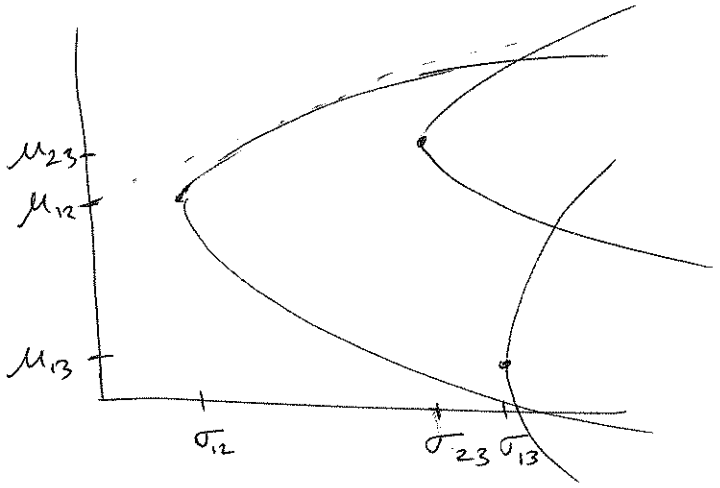
$\mu_{12} = \frac{24}{350} \sim .07$ $\mu_{13} = \frac{22}{590} \sim .037$ $\mu_{23} = \frac{1}{12} \sim .083$

$\sigma_{13}^2 = \frac{7}{2360} \cong (.05)^2$; $\sigma_{23}^2 = \frac{1}{600} \cong (.04)^2$

$\sigma_{12}^2 = \frac{3/7}{1000} \cong (.02)^2$

$A_{12}^2 = 35$; $A_{13}^2 = 59$; $A_{23}^2 = 3$

4.)



Asymptotes:

market 1,2 $\quad \mu = \mu_{12} \pm A_{12}\sigma$

etc.

5.) $$\Sigma^{-1} = \begin{pmatrix} 4 & 8 & 0 \\ 8 & 35 & -5 \\ 0 & -5 & 3 \end{pmatrix} (50)$$

min risk: (portfolio)

$$w_m = \frac{\Sigma^{-1}\mathbb{1}}{\mathbb{1}^T\Sigma^{-1}\mathbb{1}} = \begin{pmatrix} 12 \\ 38 \\ -2 \end{pmatrix} \frac{1}{48}$$

min risk line:

$$A = \left(\frac{50}{2}\right)\begin{pmatrix} 1/4 & \left(\frac{136}{40}\right) \\ \left(\frac{136}{40}\right) & 48 \end{pmatrix} \quad ; \quad A^{-1} = \frac{1}{11}\begin{pmatrix} 48 & -17/5 \\ -17/5 & 1/4 \end{pmatrix}$$

$$= 25 \begin{pmatrix} 1/4 & 17/5 \\ 17/5 & 48 \end{pmatrix}$$

min risk for return μ

$$w_\mu = \frac{\lambda_1}{2}\Sigma^{-1}\mu_K + \frac{\lambda_2}{2}\Sigma^{-1}\mathbb{1}$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = A^{-1}\begin{pmatrix} \mu \\ 1 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 48 \\ \frac{-17}{5} \end{pmatrix}\mu + \frac{1}{11}\begin{pmatrix} -17/5 \\ 1/4 \end{pmatrix}$$

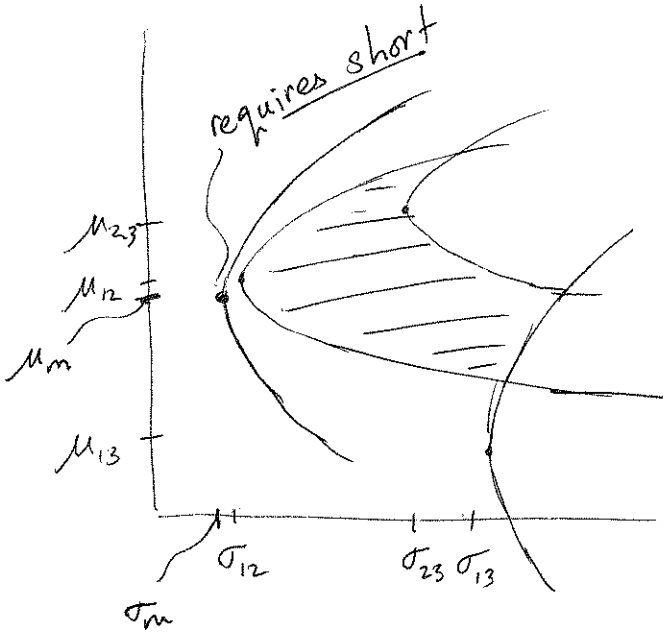
$$w_\mu = \mu\left(\frac{48}{22}\Sigma^{-1}\mu_K + \frac{-17}{110}\Sigma^{-1}\mathbb{1}\right) + \left(\frac{-17}{110}\Sigma^{-1}\mu_K + \frac{1}{88}\Sigma^{-1}\mathbb{1}\right)$$

5.) min risk & return

Return: $\mu_m = \frac{17}{240} \sim .071$ $\leftarrow \mu_m = w_m^T \mu_K$

Risk $\sigma_m^2 = w_m^T \Sigma w_m = \frac{1}{\mathbb{1}\Sigma^{-1}\mathbb{1}} = \frac{1}{2400} \cong (.02)^2$

6)



min risk requires short.

7.) Market Portfolio:

$$w_{mp} = \frac{\Sigma^{-1}(\mu_K - r\mathbb{1})}{\mathbb{1}^T \Sigma^{-1}(\mu_K - r\mathbb{1})}$$

$$\mu_K - r\mathbb{1} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \frac{1}{40}$$

$$w_{mp} = \begin{pmatrix} 8 \\ 40 \\ -8 \end{pmatrix} \frac{1}{40}$$

$$\mu_{mp} = \frac{8}{100} \; ; \quad \sigma_{mp}^2 \approx \frac{24/25}{(40)^2} \approx \left(\frac{1}{40}\right)^2$$

the market portfolio requires short.