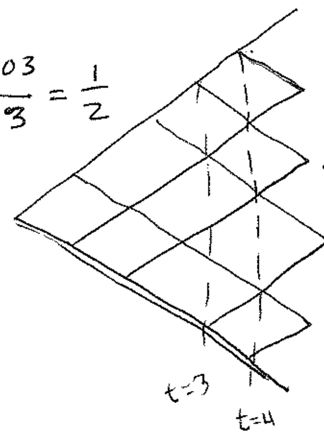
HOME WORK 7

(1) $S_0 = 100$; $r = .01$; $m_1 = .05$; $m_2 = -.03$; $X = 105$; $T = 5$

$\tilde{p}_u = \frac{.01 + .03}{.05 + .03} = \frac{1}{2}$

$\tilde{p}_d = \frac{1}{2}$



$S^{uuuuu} = 127.62 \rightarrow C(1) = 22.62$

$S^{uuuud} = 117.90 \rightarrow C(1) = 12.90$

$S^{uuudd} = 108.92 \rightarrow C(1) = 3.92$

$S^{uuddd} = 100.62 \rightarrow C(1) = 0$

$S \leq 105 \rightarrow C(1) = 0$

$S \leq 105 \rightarrow C(1) = 0$

$t = 4$

$$C_4^{dddd} = \frac{1}{1.01} \tilde{E}\{C_5 | \mathcal{F}^{dddd}\} = \frac{1}{1.01}\left\{\frac{1}{2} C_5^{dddu} + \frac{1}{2} C^{ddddd}\right\} = 0$$

$$C_4^{dddu} = \frac{1}{1.01}\left\{\frac{1}{2} C_5^{ddduu} + \frac{1}{2} C_5^{dddud}\right\} = 0$$

$$C_4^{dduu} = \frac{1}{1.01}\left\{\frac{1}{2} C_5^{dduuu} + 0\right\} = 1.94$$

$$C_4^{duuu} = \frac{1}{1.01}\left\{\frac{1}{2} C_5^{duuuu} + \frac{1}{2} C_5^{duuud}\right\} = 8.33$$

$$C_4^{uuuu} = \frac{1}{1.01}\left\{\frac{1}{2} C_5^{uuuuu} + \frac{1}{2} C_5^{uuuud}\right\} = 17.58$$

$t=3$

$C_3^{uuu} = 12.83$

$C_3^{uud} = 5.08$

$C_3^{udd} = .96$

$C_3^{ddd} = 0$

$t=2$

$C_2^{uu} = 8.87$

$C_2^{ud} = 2.99$

$C_2^{dd} = .48$

$t=1$

$C_1^{u} = 5.87$

$C_1^{d} = 1.72$

$t=0$

$$\boxed{C_0 = \frac{1}{1.01}\{\tilde{p}_u C_1^u + \tilde{p}_d C_1^d\} = 3.76}$$

2. FOR THE EUROPEAN PUT, WE MAY JUST APPLY THE C-P ~~IN~~EQUALITY.

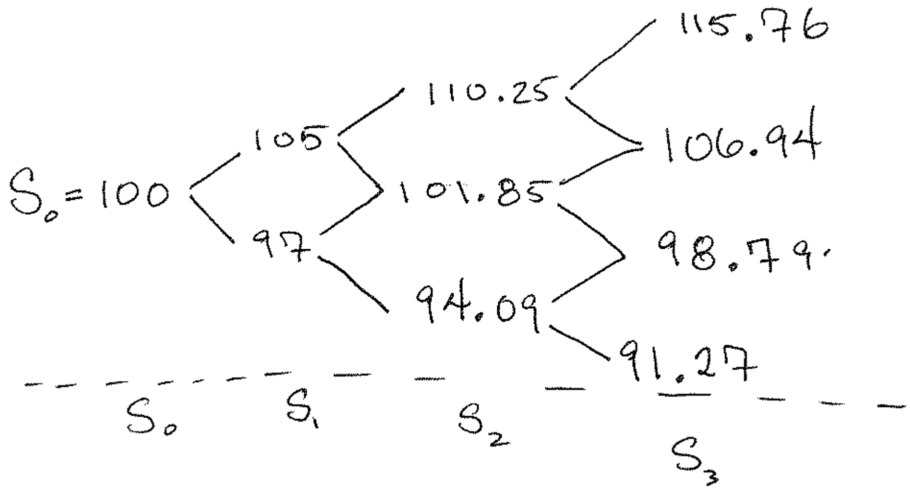
$$C_E(0) - P_E(0) = S_0 - X\,B(0,5)$$

$$P_E(0) = C_E(0) + X\,B(0,5) - S_0$$

$$= 3.76 + \frac{105}{(1.01)^5} - 100$$

$$= 3.66$$

$\underline{\underline{3}}$ FOR THIS PROBLEM, THE PRICE AT EXPIRY DEPENDS ON THE HISTORICAL VALUE OF THE STOCK.



$\underline{t=3}$

$$C^{uuu} = \left(\frac{S_1+S_2+S_3}{3} - X\right)^+ = (108.67 - 105)^+ = 3.67$$

$$C^{uud} = (107.40 - 105)^+ = 2.4$$

$$C^{udu} = \left(\frac{305.79}{3} - 105\right)^+ = 0$$

$$C^{duu} = 0$$

$$C^{udd} = C^{dud} = C^{ddu} = C^{ddd} = 0$$

$\underline{t=2}$

$$C^{uu} = \frac{1}{1.01}\left\{\tilde{p}_u C^{uuu} + \tilde{p}_d C^{uud}\right\} = 3.00$$

$$C^{ud} = 0\ ;\ C^{dd} = 0;\ C^{du} = 0$$

$t=1$

$$C^u = 1.49$$

$$C^d = 0$$

$t=0$

$$C_0 = .74$$

4

$S_0 = 200$; $r = .0002$; $m_u = .002$; $m_d = -.001$.

$X = 204$; $T = 100$

$$\tilde{p}_u = \frac{.0002 + .001}{.002 + .001} = \frac{12}{30} = .4 \; ; \; \tilde{p}_d = .6.$$

$$\tilde{q}_u = \tilde{p}_u \left(\frac{1+m_u}{1+r}\right) = .4007 \; ; \; \tilde{q}_d = .5003$$

$$k_0 = \left\lceil \frac{\log\left(\frac{204}{200\,(.999)^{100}}\right)}{\log\left(\frac{1.002}{.999}\right)} \right\rceil = \left\lceil \frac{.1198}{.002998} \right\rceil = \lceil 39.9 \rceil = 40$$

$$C_E(0) = 200\,\mathbb{P}\left(Z \geq \frac{40 - 40.07}{4.4774}\right) - \frac{X}{(1.0002)^{100}}\,\mathbb{P}\left(Z \geq \frac{40-40}{\cdots}\right)$$

$$= 200\,\underbrace{\mathbb{P}\{Z \geq -.0156\}}_{.50399} - 199.96\,\underbrace{\mathbb{P}\{Z \geq 0\}}_{1/2}$$

$$= \$.818$$

5

$N = 1{,}000$; $T = 1$ year ; $r_e = .03 \rightarrow e^r = 1.03 \Rightarrow r = .0295$

$\sigma^2 = 9 \Rightarrow m_u = \frac{rT}{N} + \sigma\sqrt{\frac{T}{N}} \cancel{\ldots} \frac{\sigma}{\sqrt{N}} \cong .09487$

$S_0 = 200$; $X = 206$] $m_d = \frac{rT}{N} + \sigma\sqrt{\frac{T}{N}}$

$$k_0 = \left\lceil \frac{\log \frac{206}{200\,(1+\frac{.03}{1000} - .09487)^{1000}}}{\log\left(\frac{1.09490}{.90516}\right)} \right\rceil = \left\lceil \frac{99.67}{.1903} \right\rceil = \lceil 523.75 \rceil$$

$$= 524$$

$$\tilde{p}_u = \frac{r\frac{T}{N} - r\frac{T}{N} + \sigma\sqrt{\frac{T}{N}}}{r\frac{T}{N} + \sigma\sqrt{\frac{T}{N}} - \left(r\frac{T}{N} - \sigma\sqrt{\frac{T}{N}}\right)} = \frac{1}{2} ; \quad \tilde{p}_d = \frac{1}{2}$$

$$\tilde{q}_u = \frac{1}{2}\left\{\frac{1 + \frac{r}{N}T + \sigma\sqrt{\frac{T}{N}}}{1 + \frac{r}{N}T}\right\} = \frac{1}{2}\left(\frac{1.0949}{1.0003}\right) = .5473$$

$$\tilde{q}_d = .4527$$

$$C_E(0) = 200\,\mathbb{P}\left(Z \geq \frac{524 - 547.3}{15.74}\right) - \frac{206}{(1.00003)^{1000}}\,\mathbb{P}\left(Z \geq \frac{524 - 500}{15.81}\right)$$

$$= 200\,\underbrace{\mathbb{P}\,(Z \geq -1.48)}_{.93056} - 199.91\,\underbrace{\mathbb{P}\,(Z \geq 1.52)}_{.06426}$$

$$= 186.11 - 12.84 = 173.26$$

The numbers in this problem are weird!
when I wrote it, I was thinking ~~~3% variance~~
~ 3% standard deviation of the return... but
I wrote $\sigma = 3 \Rightarrow 300\%$ std of return! ... should have double
checked... but that is why the call is ridiculously expensive!