Final practice

Name:____________________________________

Formulas

1.) (X,Y)

$$\int f_{X,Y}(x,y)\ dy$$

2.)

$$\mathbb{E}(XY) - \mu_X \mu_Y$$

$$\frac{c_{X,Y}}{\sigma_X \sigma_Y}$$

3.)

$$\frac{f_{X,Y}(x,y)}{f_Y(y)}$$

4.)

$$\int_{\mathbb{R}} x f_X(x|Y) dx$$

5.) Y = g(X)

$$\sum_i f_X(x_i(y)) \left| \frac{d}{dy} x_i(y) \right|$$

6.)

$$1 + q + q^2 + \cdots + q^n = \frac{1 - q^{n+1}}{1 - q}$$

7.)

$$\frac{1}{r}\left[1 - (1+r)^{-n}\right]$$

8.)

$$\left(1 + \frac{r}{m}\right)^{tm}$$

Formulas

9.)

$$w = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^T\Sigma^{-1}\mathbf{1}^T}$$

10.)

$$M = \begin{pmatrix} \mathbf{m}^T\Sigma^{-1}\mathbf{m} & \mathbf{1}^T\Sigma^{-1}\mathbf{m} \\ \mathbf{1}^T\Sigma^{-1}\mathbf{m} & \mathbf{1}^T\Sigma^{-1}\mathbf{1} \end{pmatrix}$$

11.)

$$w = \frac{\Sigma^{-1}(\mathbf{m} - r\mathbf{1})}{\mathbf{1}^T\Sigma^{-1}(\mathbf{m} - r\mathbf{1})}$$

12.) $e_1 = (1,0)^T,\ e_2 = (0,1)^T$

$$w = [(\mu\ 1)M^{-1}e_1](\Sigma^{-1}\mathbf{m}) + [(\mu\ 1)M^{-1}e_2](\Sigma^{-1}\mathbf{m})$$

13.) $\mu = \mu_r + \frac{\mu_M - \mu_r}{\sigma_M}\sigma$

14.)

$$s_0 = \frac{\sigma_2^2 - \rho_{12}\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}$$

15.)

$$(a)\ \ \mu_0 = \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2 - \rho_{12}\sigma_1\sigma_2(\mu_1 + \mu_2)}{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}; \qquad (b)\ \ \sigma_0 = \frac{\sigma_1^2\sigma_2^2 - \rho_{12}^2\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{12}}$$

16.) $\mu = \mu_0 \pm A^{-1}\sigma$

$$A^2 = \frac{\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2}{(\mu_1 - \mu_2)^2}$$

17.)

$$F(t,T) = (S(t) - \delta B(t,\tau))\frac{1}{B(t,T)}$$

18.)

$$V_X(t) = [F(t,T) - X]B(t,T)$$

19.)

$$F(0,T) = \rho\frac{B_f(0,T)}{B_d(0,T)}$$

20.) $Y_t = f(t, X_t),\ dX_t = a_t dt + b_t dW_t$

$$dY_t = (\dot{f} + \frac{1}{2}f''b_t^2)dt + f'dX_t = (\dot{f} + f'a_t + \frac{1}{2}f''b_t^2)dt + f'b_t dW_t$$

Short answer problems - write a sentence and formula for each question.

1. Define arbitrage opportunity.

2. Define Risk Neutral measure.

3. Define replicating portfolio.

Short answer problems - write a sentence and formula for each question.

4. Define completeness of a model.

5. Under what condition is the risk neutral measure unique?

6. Combining above answers and an assumption of no arbitrage justify the pricing by replicating portfolio.

7. Consider the market of 3 securities each worth \$50 at time 0. The interest over one step is $r = .1$. Determine if there is an arbitrage opportunity, and if so find it.

Security #	ω_1	ω_2	ω_3	ω_4
1	65	55	50	45
2	50	50	65	65
3	50	65	55	55

8. Consider the market of 2 securities. There are 3 possible outcomes of each security. The interest over one step is $r = 1/8$. Determine if there is an arbitrage opportunity, and if so find it. Nondiscounted returns

Return	ω_1	ω_2	ω_3
K_1	$1/8$	$-1/8$	$1/4$
K_2	$3/8$	$1/8$	0

Where $K_i = [S_i(1) - S_i(0)]/S_i(0)$.

9. Let a security with time zero value $S(0) = 100$. Time steps $S(t+1) = S(t)(1 + M_{t+1})$ where $M_{t+1} \in \{-.1, .1\}$. Interest rate $r = .05$. Consider European call with expiry $N = 3$ and strike price $X = 100$, value the call.

Solve

10. Let a security with time zero value $S(0) = 100$. Time steps $S(t+1) = S(t)(1 + M_{t+1})$ where $M_{t+1} \in \{-.1, .1\}$. Interest rate $r = .05$. Consider American put with expiry $N = 3$ and strike price $X = 100$, value the put.