Homework 3

1 For constant vectors $m \in \mathbb{R}^n$, $b \in \mathbb{R}$ and linear function $F : \mathbb{R}^n \to \mathbb{R}$ linear defined as

$$F(x) = x^t m + b$$

(here x^t denotes the transpose of x) show $\nabla F = m$

2 Again let m, b be constant vectors and let Σ be a real symmetric matrix, define a quadratic function as

$$F(x) = x^t \Sigma x + m^t x + b$$

show $\nabla F = 2\Sigma x + m$.
(Yes I switched $x^t m$ and $m^t x$ on purpose ... the order doesn't matter)

Many securities - risk and return

Suppose you are given the following securities ... $S_i(0) \equiv 100$

Ω	$S_1(1)$	$S_2(1)$	$S_3(1)$	$\mathbb{P}$
ω_1	120	135	90	1/5
ω_2	110	100	95	1/5
ω_3	100	90	110	2/5
ω_4	90	135	120	1/5

3 Find the covarance matrix Σ and expected return m of the return variables K_i for $i = 1.2.3$.

Many securities - feasible set

Suppose you have found the following Σ and m for return variables K_i, $i = 1.2.3$

$$\Sigma = \begin{pmatrix} 5 & -2 & -1 \\ -2 & 3 & -1 \\ -1 & -1 & 2 \end{pmatrix} \frac{1}{100}; \qquad m = \begin{pmatrix} .1 \\ .5 \\ 1.5 \end{pmatrix}$$

4 For each 2 security submarket $(K_1, K_2), (K_2, K_3), (K_1, K_3)$, find the minimal variance portfolio and the asymptotes of the feasible set. Graph the 3 feasible sets.

5 For the entire market, find the minimal variance portfolio, minimal variance line, and asymptotes of minimal variance line.

6 Compare these subsystems to the entire market system ie graph all systems together.

7 Suppose we add a risk free bond to the above example at return $R = .05$ Find the market portfolio and the capital market line.