1.6 Problems

Limits Properties

Question 1. Consider the limits below:

$$\lim_{x\to 1} f(x) = 2 \qquad \lim_{x\to 1} g(x) = 3 \qquad \lim_{x\to 1} h(x) = 5$$
$$\lim_{x\to 2} f(x) = -2 \qquad \lim_{x\to 2} g(x) = 7 \qquad \lim_{x\to 2} h(x) = -4$$
$$\lim_{x\to 5} f(x) = 0 \qquad \lim_{x\to 5} g(x) = -1 \qquad \lim_{x\to 5} h(x) = 1$$

Compute the following limits:

(a) $\lim_{x\to 1} f(x)g(x) = 2 \cdot 7 = 14$

(b) $\lim_{x\to 1} \frac{g(f(x))}{h(x)} = \frac{\lim_{x\to 2} g(x)}{\lim_{x\to 1} h(x)} = 7/5$

$\lim_{x\to 1} f(x) = 2$

(c) $\lim_{x\to 5} g(f(h(x))) = \lim_{x\to 1} g(f(x)) = \lim_{x\to 2} g(x) = 7.$

(d) $\lim_{x\to 2}[3f(x) + g(x)] = 3(2) + 7 = 1.$

Fractions and Cancellation

Question 2. Evaluate the following limits:

(a) $\lim_{x\to 2} \dfrac{x^2-4}{x^2+x-4}$ $= 0$

$$\frac{x^2-4}{x^2+x-4} = \frac{(x+2)(x-2)}{x^2+x-4} \longrightarrow \frac{0}{2}$$

(b) $\lim_{x\to 2} \dfrac{x^2-4}{x^2+x-6}$ $= \frac{4}{5}$

$$\frac{(x+2)(x-2)}{(x-2)(x+3)} = \frac{x+2}{x+3} \longrightarrow \frac{4}{5}$$

(c) $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{9-x}$ $= \dfrac{1}{2\sqrt{3}}$

$$\frac{\sqrt{x}-3}{(3-\sqrt{x})(3+\sqrt{x})} = \frac{1}{3+\sqrt{x}} \longrightarrow \frac{1}{2\sqrt{3}}$$

(d) $\lim_{h\to 0} \dfrac{\frac{8}{h-4}+2}{h}$ $= -\frac{1}{2}$.

$$\frac{\frac{8}{h-4}+2(h-4)}{h(h-4)} = \frac{2h}{h(h-4)} = \frac{2}{h-4} \longrightarrow -\frac{1}{2}$$

Limits with Abs Values

Question 3. Evaluate the following:

(a) $|5-1| = 4$

(b) $|1-5| = 4$

Question 4. Prove that $|a-b| = |b-a|$.

$$a-b = \begin{cases} a-b & \text{if } a-b \geq 0 \\ b-a & \text{if } a-b < 0 \end{cases}$$

similar for $b-a$

Question 5. Evaluate the limits

(a) $\lim_{x\to 1^-} \dfrac{|1-x|}{x-1} = -1$.

(b) $\lim_{x\to 1^-} \dfrac{2x(x+2)|1-x|}{x-1} = \lim_{x\to 1^-} 2x(x+2) \lim_{x\to 1^-} \dfrac{|1-x|}{x-1}$

$= -6$

Inequality Limits

Question 6. If $9 - x^2 \leq g(x) \leq 9\cos(2x)$ for all x, then find $\lim_{x\to 0} g(x)$

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$\lim_{x\to 0} 9\cos(2x) = \lim_{x\to 0} 9 - x^2 = 9$

$\therefore \lim_{x\to 0} g(x) = 9.$

Question 7. If $12x - 53 \leq f(x) \leq x^2 + 4x - 37$ for all x, then find $\lim_{x\to 4} f(x)$

$\lim_{x\to 4} 12x - 53 = -5$

$\lim_{x\to 4} x^2 + 4x - 37 = -5$

$\therefore \lim_{x\to 4} f(x) = -5$

Question 8. Evaluate the limit: $\lim_{x\to 0}\left[x^4 \sin\left(\frac{-3}{x}\right)\right]$

$-1 \leq \sin\frac{-3}{x} \leq 1$

$\therefore -x^4 \leq x^4 \sin\left(\frac{-3}{x}\right) \leq x^4$

$\therefore \lim_{x\to 0}\left(x^4 \sin\frac{-3}{x}\right) = 0.$