1.8 Problems

Remember that Algebra?

Example 1. Determine where the following functions are continuous.

(a) $f(x) = \begin{cases} 3 & \text{if } x = -1 \\ 5 & \text{if } x = -1/2 \\ \dfrac{x^2 - x - 2}{2x^2 + 3x + 1} & \text{otherwise} \end{cases}$

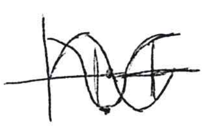
$\lim_{x \to -1/2} f(x) = DNE.$

$\lim_{x \to -1} f(x) = 3.$

$\frac{x^2-x-2}{2x^2+3x+1} = \frac{(x-2)(x+1)}{(2x+1)(x+1)} = \left(\frac{x-2}{2x+1}\right) \therefore$

$\therefore$ f is continuous @ all points but $x = -\frac{1}{2}$. Continuous on $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty$

(b) $g(x) = \dfrac{2\tan x}{\sin x + \cos x}$



$\sin x + \cos x = 0$

$@\ x = \frac{3}{4}\pi + k2\pi$

$+\ x = \frac{7}{4}\pi + k2\pi$

$k = \dots, -1, 0, 1, \dots$

$\tan x$ has asymptotes @ $x = \frac{\pi}{2} + k\pi$

continuous except @ these points.

(c) $h(x) = \sqrt{\dfrac{x^2 - x - 6}{x}} = \sqrt{\dfrac{(x-3)(x+2)}{x}}$

h is continuous on $(3, \infty)$.

Example 2. For what value of the constant c is the function

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

continuous on $(-\infty, \infty)$.

$(c+1)4 = c4 + 4 = \lim_{x \to 2^-} cx^2 + 2x = 2^3 - c2 = 8 - 2c$

$c6 = 4$

$c = \frac{2}{3}$.

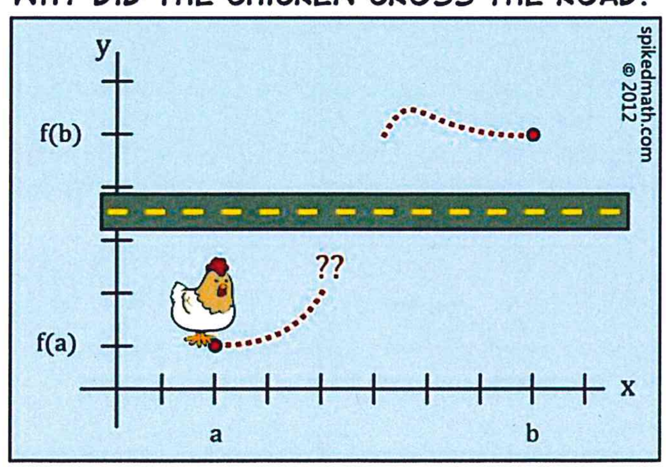
Example 3. State the Intermediate Value Theorem (from heart.)

See book

Example 4. Using the Intermediate Value Theorem explain why the chicken crossed the road (in the below picture)

answer in your own words.

WHY DID THE CHICKEN CROSS THE ROAD?



THE INTERMEDIATE VALUE THEOREM.

Example 5. Use the Intermediate Value Theorem to show that there is a root of the given equation

$\sqrt[3]{x} + x - 1 = 0.$

$f(x) = \sqrt[3]{x} + x - 1.$

$f(0) = \sqrt[3]{0} + 0 - 1 = -1.$

$f(1) = \sqrt[3]{1} + 1 - 1 = 1$

$\therefore \exists\, c \in (0, 1)$ so that $f(c) = 0.$

Example 6. Use the Intermediate Value Theorem to show that there is a root of the given equation

$\sqrt[3]{x} + x = 7.$

$g(x) = \sqrt[3]{x} + x - 7.$

$g(0) = -7$

$g(64) = \sqrt[3]{64} + 64 - 7 = 61.$

$\therefore \exists\, c \in (0, 64)$ so that

$g(c) = 0.$

Example 7. Use the Intermediate Value Theorem to show that there is a root of the given equation

$$\frac{1}{x+3} = \sqrt{x-5}.$$

$f(x) = \frac{1}{x+3} - \sqrt{x-5}$

~~f(5) ...~~ $f(5) = \frac{1}{8} - \sqrt{0} = \frac{1}{8}.$

$f(9) = \frac{1}{9+3} - \sqrt{9-5} = \frac{1}{12} - 2 = -\frac{23}{12}.$

$\therefore \exists\ c \in (5, 9)$ so that

$f(c) = 0.$

Example 8. Suppose that f to be continuous everywhere with $f(1) = 5$, $f(3) = 2$, and $f(11) = -1$. Which of the following is necessarily a true statement?

A. $f(c) = 0$ for some $c \in [1, 3]$.

B. $f(c) = 0$ for some $c \in [-1, 5]$.

C. $f(a) = f(b)$ for some $a \neq b$.

D. $f(c) = 4$ for some $c \in [1, 3]$. D

E. None of the above are true.