2.5b Problems

Tables and Graphs

Example 1. A table of values for f, g, f', and g' are given.

(a) Find the derivative of $f(g(x))$ at $x = 1$.

(b) Find the derivative of $g(f(x))$ at $x = 1$.

(c) Find the derivative of $f(f(x))$ at $x = 2$.

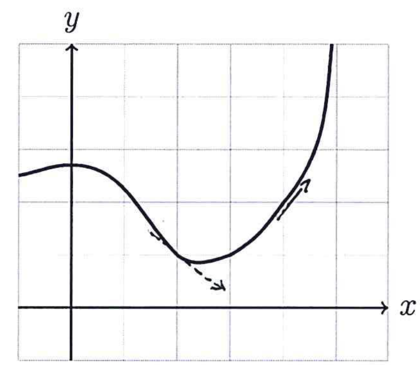
x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	3	3	0
2	1	-3	-5	6
3	4	-1	11	1

(a) $\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$; $f'(g(1))\, g'(1) = 11 * 0 = 0$

(b) $\frac{d}{dx} g(f(x)) = g'(f(x))\, f'(x)$; $g'(f(1))\, f'(1) = 6 * 3 = 18$

(c) $\frac{d}{dx} f(f) = f'(f(x))\, f'(x)$; $f'(f(2))\, f'(2) = 3 \cdot (-5) = -15$

Example 2. If f is the function whose graph is given to the right. Use the graph of f to estimate the value of each derivative:



1. $f(f(x))$ at $x = 2$.
2. $f(x^2)$ at $x = 2$.

$f(2) \approx 1$; $f'(1) \approx -1$

$f'(2) \approx -1$

(1) $\frac{d}{dx} f \circ f\ (2) = f'(f(2))\, f'(2) = (-1) * (-1) = 1$.

$f'(4) = 1$.

(2) $\frac{d}{dx} f(x^2) = f'(x^2)\, 2x$; $f'(2^2) \cdot 2 \cdot 2 = 1 \cdot 4 = 4$.

Standard Problems

Example 3. Find the derivatives of the following functions:

(a) $f(x) = \dfrac{3}{x}\cos^{-4} x$

$$f' = 3\,\frac{x(-4)\cos^{-5}x(-\sin x) - \cos^{-4}x}{x^2}$$

(b) $g(x) = ((4x + x^3)^{-2} + 3x)^4$

$$g' = 4\left((4x+x^3)^{-2}+3x\right)^3\left((4x+x^3)^{-2}+3x\right)'$$

$$= 4\left((4x+x^3)^{-2}+3x\right)^3\left(-2(4x+x^3)^{-3}(4+3x^2)+3\right)$$

(c) $h(t) = \sin(\cos(\tan(2t)))$

$$h' = \left(\sin(\cos(\tan(2t)))\right)' = \cos(\cos(\tan(2t)))\,(\cos(\tan(2t))$$

$$= \cos\cos\tan(2t)\,(-\sin\tan 2t)\,(\tan 2t)'$$

$$= -\left(\cos(\cos(\tan(2t))\right)\left(\sin(\tan 2t)\right)\sec^2 2t)2.$$

Example 4. Find an equations of the tangent line to the curve at the given point:

(a) $f(x) = (1+2x)^{10}$ at $x = 0$.

$$f' = 10\,(1+2x)^9\,2.$$

$$f(0) = 1$$

$$f'(0) = 20$$

$$\frac{y-1}{x-0} = 20. \Rightarrow \quad y = 1 + 20x.$$

(b) $g(x) = \sqrt{1+x^3}$ at $x = 2$

$$g' = \frac{1}{3}(1+x^3)^{-2/3}\,(x^3)' = \frac{1}{3}\,(1+x^3)^{-2/3}\,3x^2$$

$$= \frac{x^2}{(1+x^3)^{2/3}}$$

$$g(2) =$$

(c) $h(x) = \sin x + \sin^2 x$ at $(0,0)$

$$h' = \cos x + 2\sin x \cos x \quad @\ (0,0).$$

$$h(0) = 0.$$

$$h'(0) = 1. \qquad \frac{y-0}{x-0} = 1$$

$$\Rightarrow y = x.$$

Non-Standard (Fun) Problems

Example 5. If $h(x) = \sqrt{4 + 3f(x)}$ where $f(1) = 7$ and $f'(1) = 4$, find $h'(1)$.

$$h'(x) = \left((4+3f(x))^{1/2}\right)' = \frac{1}{2}\,(4+3f(x))^{-\frac{1}{2}}\,(3f(x))'$$

$$= \frac{3\,f'(x)}{2\sqrt{4+3f(x)}}$$

$$h'(1) = \frac{3\cdot 4}{2\sqrt{4+3\cdot 7}} = \frac{3\cdot 4}{2\cdot 5} = \frac{6}{5}.$$

Example 6. Write $|x| = \sqrt{x^2}$ and use the chain rule to prove that $\dfrac{d}{dx}|x| = \dfrac{x}{|x|}$

Example 7. If $f(x) = |\sin x|$, find $f'(x)$. Where is f not differentiable?