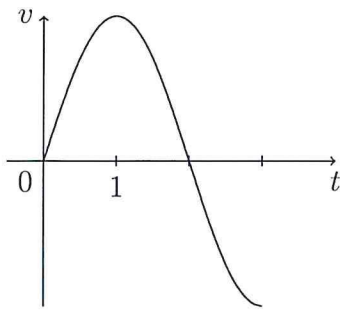
# 2.7 Problems

## Graphs

**Example 1.** Graphs of the velocity functions of two particles are shown, where $t$ is measured in seconds. When is each particle speeding up? When is it slowing down? Explain.

(a)

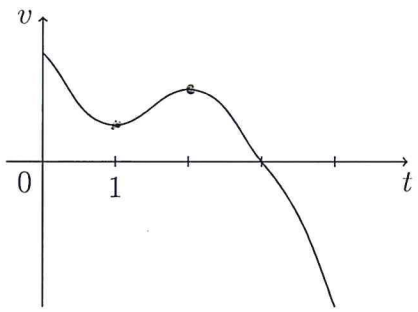



Particle is still @
$t=0$ & $t=2$
It reaches greatest velocity @ $t=1$ & $t=3$

Speeding up:
$(0,1) \cup (2,3)$

slowing down:
$(1,2)$

(b)

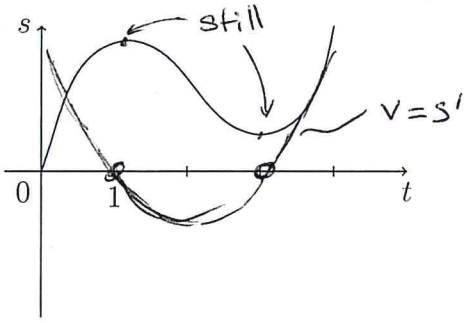



Slowing down:
$(0,1) \cup (2,3)$
graph is approaching zero.

Speeding up:
$(1,2) \cup (3,4)$

**Example 2.** Graphs of the position functions of two particles are shown, where $t$ is measured in seconds. When is each particle speeding up? When is it slowing down? Explain.

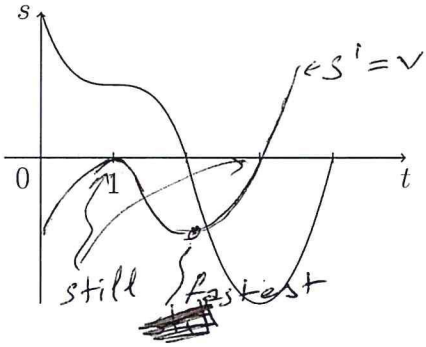
(a)




Slowing down:
$(0,1) \cup (2,3)$

Speeding up:
$(1,2) \cup (3,4)$

(b)




Slowing down: $(0,1) \cup (2,3)$

Speeding up: $(1,2) \cup (3,4)$.

## Standard Problems

**Example 3.** A particle moves according to the position function $s(t) = \dfrac{t}{(1+t^2)}$ on the interval $t \geq 0$, where $t$ is measured in seconds and $s$ in feet.

(a) Find the velocity at time $t$.

$$V(t) = S' = \frac{(t)'(1+t^2) - t(1+t^2)'}{(1+t^2)^2} = \frac{1+t^2-2t^2}{(1+t^2)^2} = \frac{1-t^2}{(1+t^2)^2}$$

(b) When is the particle at rest.

particle is @ rest when $V(t) = 0 \iff 1-t^2 = 0$

ie $t^2 = 1 \Rightarrow t = 1$.

$t \geq 0$

(c) When is the particle moving in the positive direction?

particle is moving in positive direction when $v(t) > 0$.

$V(t) > 0 \Rightarrow 1-t^2 > 0 \Rightarrow 0 \leq t < 1$.

$t \geq 0$

(d) Find the total distance traveled in first 8 seconds

moving in positive direction on $(0,1)$

" " negative " " $(1,\infty)$

movement from $(0,1) = |S(1) - S(0)| = 1/2$.

movement during $(1,8) = |S(1) - S(8)| = 1/2 - \frac{8}{65}$

$\therefore$ total movement $= 1 - \frac{8}{65}$.

(**Example 3** continued) Recall $s(t) = \dfrac{t}{(1+t^2)}$ on the interval $t \geq 0$.

(e) Calculate the acceleration of the particle at time $t$. $a(t) = v'(t) \equiv$ acceleration.

$$v(t) = \frac{1-t^2}{(1+t^2)^2}$$

$$v'(t) = \frac{(1+t^2)^2(1-t^2)' - (1-t^2)\left((1+t^2)^2\right)'}{(1+t^2)^2}$$

$$= \frac{-2t(1+t^2)^2 - (1-t^2)\,2(1+t^2)\,2t}{(1+t^2)^4}$$

(f) When is the particle speeding up? particle speeding up when $a(t) > 0$ & $v(t) > 0$ or $a(t) < 0$ & $v(t) < 0$

$$v' = \frac{-2t(1+t^2)^2 - (1-t^4)\,4t}{(1+t^2)^4}$$

$$= \frac{2t(t^2-3)(t^2+1)}{(1+t^2)^4} = 2t\,\frac{(t^2-3)}{(1+t^2)^3}$$

$v' > 0$ on $t > \sqrt{3}$ $\qquad$ $v > 0$ on $0 < t < 1$

$v' < 0$ on $0 < t < \sqrt{3}$ $\qquad$ $v < 0$ on $1 < t$.

$\therefore$ $v' < 0$ & $v < 0$ on $1 < t < \sqrt{3}$.

**Example 4.** A ball is thrown vertically upward on planet X with an initial velocity of 10 meters per second. Its height after $t$ seconds is given by $h(t) = -at^2 + 10t + 1$

(a) Find the value of $a$ if the ball reaches its maximum height after 5 seconds

The ball reaches its max height @ $t$ s.t. $h'(t) = 0$

$h'(t) = -2at + 10$.

$0 = h'(5) = -10a + 10 \Rightarrow a = 1$.

(b) What is the balls maximum height?

$h(5) = -5^2 + 10 \cdot 5 + 1 = 26$.

(c) When will the ball hit the ground? The ball hits the ground @ $t > 0$ s.t. $h(t) = 0$

use quadratic formula: $t = \frac{-10 \pm \sqrt{100 + 4}}{-2} = 5 \mp \sqrt{26}$

require $t > 0$ $\therefore$ Ball hits ground @ $5 + \sqrt{26}$.

(d) How fast is the ball traveling when it is 2 meters above the ground on the way down?

$h(t) = 2$ w/ $t > 0$.

$h(t) = 2 = -t^2 + 10t + 1$; speed is: $h'(t_0) = -2(5 + \sqrt{24}) + 10$

quad formula: $= -2\sqrt{24}$

$t_0 = 5 \mp \sqrt{24}$