2.8 Problems

Level 1 Problems

Example 1. The length of a rectangle is increasing at a rate of 4 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 5 cm and the width is 6 cm:

(a) how fast is the area of the rectangle increasing?

Area of rectange $A = l * w$

$$(l, w) = (5, 6) \quad ; \quad (l', w') = (4, 3)$$

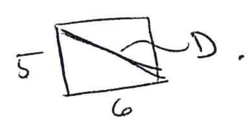
$$A' = (lw)' = l'w + lw' = 4 \cdot 6 + 5 \cdot 3$$

(b) how fast is the perimeter of the rectangle increasing?

Perimeter : $P = 2l + 2w$.

$$P' = 2l' + 2w' = 2(4+3) = 14$$

(c) how fast is the diagonal of the rectangle increasing?



$$D^2 = l^2 + w^2$$

$$D^2 = 5^2 + 6^2 = 61 \Rightarrow D = \sqrt{61}$$

$$2DD' = \frac{\partial}{\partial t} D^2 = \frac{\partial}{\partial t}(l^2 + w^2) = 2ll' + 2ww'$$

$$D' = \frac{ll' + ww'}{D} = \frac{5 \cdot 4 + 6 \cdot 3}{\sqrt{61}}$$

Example 2. If $x^2 + y^2 + z^2 = 9$, $dx/dt = 5$, and $dy/dt = 4$ find dz/dt when $(x, y, z) = (2, 2, 1)$.

$$0 = \frac{d}{dt} 9 = \frac{d}{dt}(x^2+y^2+z^2) = 2xx' + 2yy' + 2zz'$$

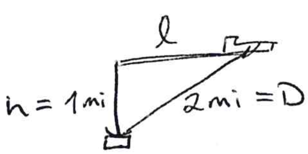
$$= 2\cdot 2\cdot 5 + 2\cdot 2\cdot 4 + 2\cdot 1\cdot z'$$

pluggingin vals.

$$\therefore \quad \underline{z' = -18}$$

Level 2 Problems

Example 3. A plane flying horizontally at an altitude of 1 mi and a speed of 300 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.



$$1^2 + \ell^2 = 2^2 \Rightarrow \ell = \sqrt{3}.$$

$$(\ell, h, D) = (\sqrt{3}, 1, 2).$$

$$\ell' = 300 \;;\; h' = 0 \;;\; D' = ?$$

$$D^2 = h^2 + \ell^2$$

$$2\cdot 2\cdot D' = 2DD' = \underbrace{2hh'}_{0} + 2\ell\ell' = 2\ell\ell' = 2\sqrt{3}\; 300$$

$$D' = \sqrt{3}\cdot 150$$

Example 4. A particle is moving along a hyperbola $xy = 8$. As it reaches the point $(4, 2)$, the $y-$coordinate is decreasing at a rate of 3 cm/s. How fast is the $x-$coordinate of the point changing at that instant?

$$(x, y) = (4, 2)$$

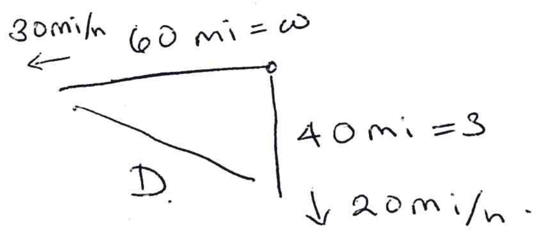
$$(x', y') = (?\,, -3)$$

$$8 = xy$$

$$0 = \frac{d}{dt} 8 = \frac{d}{dt} xy = x'y + xy' = x' \cdot 2 + 4(-3)$$

$$\underline{\underline{x' = 6.}}$$

Example 5. Two cars start moving from the same point. One travels south at 20 mi/h and the other travels west at 30 mi/h. At what rate is the distance between the cars increasing two hours later?



@ 2hrs cars have traveled
(i) 60 mi west
(ii) 40mi south.

$$D^2 = w^2 + s^2$$

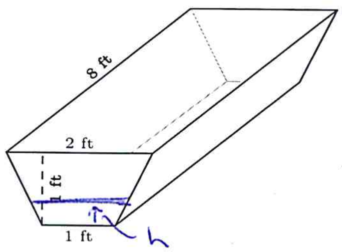
$$2DD' = 2ww' + 2ss' = 2(60)(30) + 2(40)(20)$$

$$D = \sqrt{60^2 + 40^2} = \sqrt{52} \times 10$$

$$D' = \frac{260}{\sqrt{52}}$$

Level 3 Problem

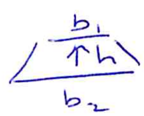
Example 6. A trough is 8 ft long and has a cross section of an isosceles trapezoid with base of 1 ft, height of 1 ft, and top of 2 ft (see the picture below). If the trough is being filled with water at the rate of 3 ft^3/min. how fast is the water level rising when the water is 6 inches deep?



area of Trapezoid: $\frac{b_1+b_2}{2} \cdot h = A$

Volume of cylinder: $V = A * l$.

$l = 8ft$



1 base of trapezoid is 1 ft.

2nd base is $(1+h)$ ft

Volume:

$$V = \left(\frac{2+h}{2} \cdot h\right) 8 = 4(2+h)h\,(ft)^3.$$

$$3ft^3/min = V' = (4h + 4(2+h))\,(ft)^3/min * h'$$

$h = 6in = .5$ ft

$$3 = (2 + 8 + 2)h' = 12\,h'$$

$\Rightarrow \quad h' = \frac{1}{4}$ ft/min.